WEST VIRGINIA
SECRETARY OF STATE

NATALIE E. TENNANT

ADMINISTRATIVE LAW DIVISION

Form #5

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OFFICE WEST VIRGINIA
SECRETARY OF STATE

NOTICE OF AGENCY ADOPTION OF A PROCEDURAL OR INTERPRETIVE RULE OR A LEGISLATIVE RULE EXEMPT FROM LEGISLATIVE REVIEW

AGENCY: West Virginia Board of Education TITLE NUMBER: 126

CITE AUTHORITY: W. Va. Constitution, Article XII, §2, W. Va. Code §18-2-5 and §18-9A-22

RULE TYPE: PROCEDURAL ______ INTERPRETIVE ______

EXEMPT LEGISLATIVE RULE X
CITE STATUTE(s) GRANTING EXEMPTION FROM LEGISLATIVE REVIEW
W.Va. Code §§29A-3B-1, et seq.; *W.Va. Board of Education v. Hechler*, 180 W.Va. 451; 376 S.E.2d 839 (1988).

AMENDMENT TO AN EXISTING RULE: YES ____ NO X

IF YES, SERIES NUMBER OF RULE BEING AMENDED: ______

TITLE OF RULE BEING AMENDED: ______

IF NO, SERIES NUMBER OF NEW RULE BEING PROPOSED: 44BB

TITLE OF RULE BEING PROPOSED: Next Generation Content Standards and Objectives for Mathematics in West Virginia Schools (2520.2B)

THE ABOVE RULE IS HEREBY ADOPTED AND FILED WITH THE SECRETARY OF STATE. THE EFFECTIVE DATE OF THIS RULE IS August 15, 2011.

[signature]

Charles K. Heinlein
Deputy State Superintendent of Schools

EXECUTIVE SUMMARY
WEST VIRGINIA DEPARTMENT OF EDUCATION

Policy 2520.2B – Next Generation Content Standards and Objectives for Mathematics in West Virginia Schools

Background: The Next Generation Content Standards and Objectives in West Virginia Schools are aligned to the Common Core State Standards for Mathematics, the culmination of an extended, broad-based effort to fulfill the change issued by the states to create the next generation of K-12 standards in order to help ensure that all students are college and career ready in literacy no later than the end of high school. The Common Core State Standards for Mathematics, the product of work led by the Council of Chief State School Officers (CCSSO) and the National Governors Association (NGA), builds on the foundation laid by the states in their decades-long work on crafting high-quality education standards. In May 2010, the West Virginia Board of Education adopted the Common Core State Standards for Mathematics, shortly thereafter, 85 classroom teachers and representatives of Higher Education faculty began a deep study of this work and place the content of these Standards into the West Virginia Framework. This group of West Virginia educators found the standards to be research and evidence-based, aligned with college and work expectations, rigorous, and internationally benchmarked. A particular standard was included in the document only when the best available evidence indicated that its mastery was essential for college and career readiness in a twenty-first century, globally competitive society.
The standards are substantially more focused and coherent in order to improve mathematics achievement. The development of these standards' mathematical knowledge, skill and understanding develop over time. These standards define what a student should know and do to be globally competitive in the 21st century.

Proposals: Adopt W. Va. 126CSR44BB, Policy 2520.2B, Next Generation Content Standards and Objectives for Mathematics in West Virginia Schools, along with new performance descriptors to be effective as follows: kindergarten August 15, 2011; first grade July 1, 2012; second grade July 1, 2013; third through twelfth July 1, 2014. These align with the board adopted Common Core State standards and have been placed in the West Virginia framework. These will assure a rigorous and thorough curriculum, which will assure that West Virginia students are college and career ready at the end of grade twelve.

Impact: The proposed adoption of the Next Generation Content Standards and Objectives for Mathematics in West Virginia Schools will provide stakeholders with the curriculum as well as performance at each graced level from kindergarten through twelfth grade. Students curriculum will be aligned with other states that have adopted the Common Core State Standards and will assure that students will be college and career ready when they exit grade twelve.

Response to Comments: Nineteen comments were received. Twelve comments related to the rigor of the standards being too difficult for students and the organization of the High School courses. Two comments praised the high level of rigor of the standards. One comment related to elementary performance descriptors in grades 2, 3 and 4 helped to improve clarity. Three comments were related to the sequencing of the high school courses. One comment was related to formatting. Ten revisions were made regarding format. Six revisions were made based on two of the comments (performance descriptors and formatting). The complete log is attached.

Educators Consulted about Revision of Policy 2520.2B

Internal:
Office of Instruction
- Carla Williamson, Executive Director
- Lou Maynus, Mathematics Coordinator
- Lynn Baker, Math/Science Partnership Coordinator

Office of Assessment and Accountability
- Sonya White, Assistant Director
- Terri Sappington, Coordinator

Office of Title I
- John Ford, Coordinator

Office of Special Programs
- Mary Pat Farrell, Coordinator

External:
Mathematics Teachers
- Janet Bowland, Fayette
- Allison Miller, Roane
- Dana Yokum, Pendleton
- Billie Jo Falcon, Ohio
- Margaret Ripley, Marshall
- Tonya Hatcher, Mingo
- Eileen Wally, Ohio
- Rachel Hull, Putnam
- Ann Lienhardt, Marion
- Mary Ann Gaston, Marion
- Myrtle Holland, Berkeley
- Jeremy Knight, Berkeley
- Cindy Burke, Marshall
- Sheila Ruddle, Pendleton
- Jerry Pomeroy, Greenbrier
- Kim Tickett, Cabell
- Joy McCutcheon, Kanawha
- Neil Reger, Upshur
- Cheryl Reger, Upshur
- Mike Brown, Nicholas
- Holly Branch, Berkeley
- Debbie Seldomridge, Mineral
- Allan Meck, Hampshire

Math Coaches:
- Mary Perdue, McDowell
- Susan Barrett, Nicholas
- Janie Merendino, Marion
- ShellyPrince, Raleigh
- Jeanie Brown, Fayette
- Joyce Evans, Marion
- Susan Naylor, Wood
  Melanie Meck, Hampshire

RESA

- Judy Pomeroy, Professional Development Coordinator, RESA IV

Higher Education

- Dr. Karen Mitchell, Marshall University
- Dr. Mary Ann Clark, West Virginia University
- Dr. Gary Seldomridge, Potomac State
- Dr. Laura Pyzdrowski, West Virginia University
- Dr. Mike Mays, West Virginia University
- Dr. Cecilia Fizer, Concord University
- Dr. Al Edward, West Virginia University at Parkersburg
- Dr. Jean Bolyard, Fairmont State University

TITLE 126
LEGISLATIVE RULE
BOARD OF EDUCATION

SERIES 44BB
Next Generation Content Standards and Objectives for Mathematics In West Virginia Schools (2520.2B)

§126-44BB-1. General.

1.1. Scope. – W. Va. 126CSR42, West Virginia Board of Education Policy 2510, Assuring the Quality of Education: Regulations for Education Programs (hereinafter Policy 2510) provides a definition of a delivery system for, and an assessment and accountability system for, a thorough and efficient education for West Virginia public school students. Policy 2520.2B defines the content standards objectives for mathematics as required by Policy 2510.

1.2. Authority. - W. Va. Constitution, Article XII, §2, W. Va. Code §18-2-5 and §18-9A-22.

1.3. Filing Date. – July 15, 2011.

1.4. Effective Date. – Kindergarten August 15, 2011; First Grade July 1, 2012; Second Grade July 1, 2013; Third through Twelfth July 1, 2014.

1.5. Repeal of former rule. – None. This is a new policy.

§126-44BB-2. Purpose.

2.1. This policy defines the content standards and objectives for the program of study required by Policy 2510 in mathematics.

§126-44BB-3. Incorporation by Reference.

3.1. A copy of the Next Generation Content Standards and Objectives for Mathematics in West Virginia Schools is attached and incorporated by reference into this policy. Copies may be obtained in the Office of the Secretary of State and in the West Virginia Department of Education, Office of Instruction.

§126-44BB-4. Summary of the Content Standards and Objectives.

4.1. The West Virginia Board of Education has the responsibility for establishing high quality standards pertaining to all educational standards pertaining to all education programs (W. Va. Code §18-9A-22). The content standards and objectives provide a focus for teachers to teach and students to learn those skills and competencies essential for future success in the workplace and further education. The document includes content standards for mathematics; an explanation of terms; objectives that reflect a rigorous and challenging curriculum; and performance descriptors.

Introduction

The Next Generation Content Standards and Objectives for Mathematics in West Virginia Schools are aligned to the Common Core State Standards for Mathematics, the culmination of an extended, broad-based effort to fulfill the charge issued by the states to create the next generation of K-12 standards in order to help ensure that all students are college and career ready no later than the end of high school. The Common Core State Standards for Mathematics, the product of work led by the Council of Chief State School Officers (CCSSO) and the National Governors Association (NGA), builds on the foundation laid by the states in their decades-long work on crafting high-quality education standards. In May 2010, the West Virginia Board of Education adopted the Common Core State Standards for Mathematics; shortly thereafter, 85 classroom teachers and representatives of Higher Education faculty began a deep study of this work and placed the content of these Standards into the West Virginia Framework. This group of West Virginia educators found the standards to be research and evidence-based, aligned with college and work expectations, rigorous, and internationally benchmarked. A particular standard was included in the document only when the best available evidence indicated that its mastery was essential for college and career readiness in a twenty-first-century, globally competitive society.

For over a decade, research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country. To deliver on the promise of common standards, the standards must address the problem of a curriculum that is "a mile wide and an inch deep." These Standards are a substantial answer to that challenge. It is important to recognize that "fewer standards" are no substitute for focused standards. Achieving "fewer standards" would be easy to do by resorting to broad, general statements. Instead, these Standards aim for clarity and specificity. Assessing the coherence of a set of standards is more difficult than assessing their focus. William Schmidt and Richard Houang (2002) have said that content standards and curricula are coherent if they are articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchical nature of the disciplinary content from which the subject matter derives. That is, what and how students are taught should reflect not only the topics that fall within a certain academic discipline, but also the key ideas that determine how knowledge is organized and generated within that discipline. This implies that to be coherent, a set of content standards must evolve from particulars (e.g., the meaning and operations of whole numbers, including simple math facts and routine computational procedures associated with whole numbers and fractions) to deeper structures inherent in the discipline. These deeper structures then serve as a means for connecting the particulars (such as an understanding of the rational number system and its properties). These Standards endeavor to follow such a design, not only by stressing conceptual understanding of key ideas, but also by continually returning to organizing principles such as place value or the properties of operations to structure those ideas.

The sequence of topics and performances that is outlined in a body of mathematics standards must also respect what is known about how students learn. In recognition of this, the development of these Standards began with research-based learning progressions detailing what is known today about how students' mathematical knowledge, skill, and understanding develop over time. In the early grades there is greater focus and coherence. Mathematics experiences in early childhood settings should concentrate on (1) number, which includes whole number, operations, and relations, and (2) geometry, spatial relations, and measurement, with more mathematics learning time devoted to number than to other topics. Mathematical process goals should be integrated in these content areas.

Explanation of Terms

Content Standards are broad statements that define the knowledge, skills and understanding that all students must demonstrate in a content area at the end of the kindergarten through college career readiness sequence of study.

Clusters are groups of the objectives that define the expectations students must demonstrate to be college and career ready.

Objectives are incremental steps toward accomplishment of content standards. Objectives are listed by grade level and are organized around the clusters and content standards. Objectives build across grade levels as students advance in their knowledge and skills.

Performance Descriptors describe in narrative format how students demonstrate achievement of the content standards. Line breaks within the narrative format indicate clusters of concepts and skills. West Virginia has designed five performance levels: distinguished, above mastery, mastery, partial mastery, and novice. Performance Descriptors serve two functions. Instructionally, they give teachers more information about the level of knowledge and skills students need to acquire. Performance levels and descriptors are also used to categorize and explain student performance on statewide assessment instruments.

Distinguished: A student at this level has demonstrated exemplary performance. The work shows a distinctive and sophisticated application of knowledge and skills that go beyond course or grade level applications.

Above Mastery: A student at this level has demonstrated effective performance and exceeds the standard. The work shows a thorough and effective application of knowledge and skills.

Mastery: A student at this level has demonstrated adequate knowledge and skills that meet the standard. The work is accurate, complete and fulfills all requirements. The work shows solid academic performance at the course or grade level.

Partial Mastery: A student at this level has demonstrated limited knowledge and skills toward meeting the standard. The work shows basic but inconsistent application of knowledge and skills characterized by errors and/or omissions. Performance needs further development.

Novice: A student at this level has demonstrated minimal fundamental knowledge and skills needed to meet the standard. Performance at this level is fragmented and/or incomplete and needs considerable development.

Numbering of Standards
The number for each standard is composed of three parts, each part separated by a period:

- the content area code is M for Mathematics,
- the grade level, and
- the standard.

Illustration: M.3.G refers to the third grade Geometry Mathematics standard.

The Mathematics Standards are listed below:

CC	Counting and Cardinality
OA	Operations and Algebraic Thinking
NBT	Number and Operations in Base Ten
G	Geometry
MD	Measurement and Data
NF	Number and Operations – Fractions
NS	The Number System
EE	Expressions and Equations
SP	Statistics and Probability
RP	Ratio and Proportional Relationships
F	Functions
RBQ	Relationships Between Quantities
LER	Linear and Exponential Relationships
RWE	Reasoning with Equations
DST	Descriptive Statistics
CPC	Congruence, Proof and Constructions
CAG	Connecting Algebra with Geometry through Coordinates
ENS	Extending the Number System
QFM	Quadratic Functions and Modeling
AOP	Applications of Probability
STP	Similarity, Right Triangles, and Trigonometry
C	Circles With and Without Coordinates
IC	Inferences and Conclusions from Data
PR	Polynomials, Radical Relationship
TF	Trigonometry of General Triangles and Trigonometric Functions
MM	Mathematical Modeling

Numbering of Objectives
The numbering of objectives is composed of four parts, each part separated by a period:

- the content area code is M for Mathematics,
- the grade level,
- the standard, and
- the objective.

Illustration: M.K.CC.4 refers to the fourth objective in the standard Counting and Cardinality.

Numbering of Performance Descriptors
The number for each group of four performance descriptors is composed of three parts, each part separated by a period:

- the content area (M for Mathematics),
- the letters PD are for Performance Descriptors,
- the grade level, and
- the standard number.

Illustration: M.PD.4.NBT refers to Mathematic performance descriptors for the fourth grade Number and Operations in Base Ten Standard.

Unique Electronic Numbers (UENs)
Unique Electronic Numbers (or UENs) are numbers that help to electronically identify, categorize and link, specific bits of information. Once Policy 2520.2B is available on the Web, each standard, each cluster, each objective and each group of five performance descriptors will have a Unique Electronic Number (UEN) that will always remain the same.

The codes printed in Policy 2520.2B form the basis of the UENs. The only additional set of numbers that will be added to each code to formulate its UEN will be a prefix that indicates the year and month that a particular version of Policy 2520.2B is approved by the State Board of Education.

The prefix for the UENs for each content area in Policy 2520.2B is noted at the top of each page containing standards, clusters, objectives and performance descriptors. As sections of Policy 2520.2B are revised, UENs will be changed to reflect the new approval date.

UENs (Unique Electronic Numbers) facilitate implementation of WV Standards into electronic formats such as databases and XML Files. The WV Department of Education encourages everyone who is going to use the Next Generation Content Standards for English Language Arts in any kind of electronic distribution, alignment, or software development to use the UENs so that all efforts can be cross-referenced and there is consistency across initiatives.

Mathematics – Policy 2520.2B

The West Virginia Next Generation Standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The Standards set grade-specific standards but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. For example, for students with disabilities reading should allow for use of Braille, screen reader technology, or other assistive devices, while writing should include the use of a scribe, computer, or speech-to-text technology. In a similar vein, speaking and listening should be interpreted broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the Standards do provide clear signposts along the way to the goal of college and career readiness for all students

The Standards begin with eight Standards for Mathematical Practice.

Mathematics: Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

MP1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize* - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

MP3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies.

Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

MP4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MP8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$ and $(x - 1)(x^3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development and student achievement in mathematics.

Mathematics – Kindergarten

In Kindergarten, instructional time should focus on two critical areas: (1) representing and comparing whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

1. Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as $5 + 2 = 7$ and $7 - 2 = 5$. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets or counting the number of objects that remain in a set after some are taken away.

2. Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade K	**Mathematics**			
Standard	Counting and Cardinality			
Performance Descriptors M.PD.K.CC				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Kindergarten students at the distinguished level in mathematics: write and represent numbers beyond 100;	Kindergarten students at the above mastery level in mathematics: count beyond 100 by ones, twos, fives, and tens, and write and represent numbers beyond 20;	Kindergarten students at the mastery level in mathematics: count to 100 by ones and tens, count forward from a given number, write and represent numbers 0 to 20 using concrete objects;	Kindergarten students at the partial mastery level in mathematics: count to 29, read, write, and represent numbers to 19 using concrete objects;	Kindergarten students at the novice level in mathematics: count to 10, read, copy and represent some numbers with objects;

use multiple strategies to count how many objects when given different arrangements and structure and justify the strategies used;	use a strategy to count how many objects when given different arrangements and structures;	know that the last number said tells the number of objects counted and that each successive number is one more, tell how many when given different arrangements and structures of up to twenty objects;	know that the last number said tells the number of objects counted and tells how many given different arrangements and structures with up to ten objects;	count some objects in different arrangements;
compare two two-digit numbers using multiple strategies and justify the use of greater than, less than or equal to symbols.	use multiple strategies to compare groups of objects and numerals using greater than, less than or equal to symbols.	use matching and counting strategies to identify groups of objects as greater than, less than, or equal to the number of objects in another group and compare values of written numerals between 1 and 10.	use matching and counting strategies to name groups of objects and compare some numerals between 1 and 10.	use a counting strategy to compare groups of objects to identify the group with more objects.

Cluster	**Know Number Names and the Count Sequence**
Objectives	**Students will**
M.K.CC.1	count to 100 by ones and by tens.
M.K.CC.2	count forward beginning from a given number within the known sequence (instead of having to begin at 1).
M.K.CC.3	write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).
Cluster	**Count to Tell the Number of Objects**
Objectives	**Students will**
M.K.CC.4	understand the relationship between numbers and quantities; connect counting to cardinality a. when counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object, b. understand that the last number name said tells the number of objects counted and the number of objects is the same regardless of their arrangement or the order in which they were counted, c. understand that each successive number name refers to a quantity that is one larger.
M.K.CC.5	count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.
Cluster	**Compare Numbers**
Objectives	**Students will**
M.K.CC.6	identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.
M.K.CC.7	compare two numbers between 1 and 10 presented as written numerals.

Grade K	**Mathematics**			
Standard	Operations and Algebraic Thinking			
Performance Descriptors M.PD.K.OA				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Kindergarten students at the distinguished level in mathematics: fluently add and subtract with and without representation and write equations to solve word problems.	Kindergarten students at the above mastery level in mathematics: represent addition and subtraction with and without symbols, create and solve word problems with and without objects or drawings, and write equations.	Kindergarten students at the mastery level in mathematics: represent addition and subtraction within ten (fluently to five), solve word problems, and decompose numbers.	Kindergarten students at the partial mastery level in mathematics: represent addition with objects, fingers, drawings, or role play, represent addition word problems using objects or drawings, and decompose numbers to 5.	Kindergarten students at the novice level in mathematics: represent "how many" with objects, drawings, or role play.

Cluster	**Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.**
Objectives	Students will
M.K.OA.1	represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions or equations.
M.K.OA.2	solve addition and subtraction word problems and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
M.K.OA.3	decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
M.K.OA.4	for any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
M.K.OA.5	fluently add and subtract within 5.

Grade K	**Mathematics**			
Standard	Number and Operation in Base Ten			
Performance Descriptors M.PD.K.NBT				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Kindergarten students at the distinguished level in mathematics: justify the decomposition of two-digit numbers.	Kindergarten students at the above mastery level in mathematics: compose and decompose two-digit numbers greater than 19.	Kindergarten students at the mastery level in mathematics: compose and decompose numbers from 11 - 19 using place value with objects, drawings, or equations.	Kindergarten students at the partial mastery level in mathematics: compose and decompose some numbers using place value and objects.	Kindergarten students at the novice level in mathematics: verbalize the counting sequence above ten and decompose numbers to ten with objects.

Cluster	**Work with Numbers 11-19 to Gain Foundation for Place Value**
Objectives	**Students will**
M.K.NBT.1	compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation, e.g., 18 = 10 + 8; understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Grade K	**Mathematics**			
Standard	Measurement and Data			
Performance Descriptors M.PD.K.MD				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Kindergarten students at the distinguished level in mathematics: represent and design shapes using measurable attributes; justify the classification of objects and show representation.	Kindergarten students at the above mastery level in mathematics: organize shapes according to measureable attributes; categorize and describe groups of objects.	Kindergarten students at the mastery level in mathematics: describe and compare measureable attributes using vocabulary such as more/less, taller/shorter, etc.; classify, count, and sort objects equal to or less than ten.	Kindergarten students at the partial mastery level in mathematics: recognize some measureable attributes; counts objects by given attributes.	Kindergarten students at the novice level in mathematics: match objects by a given attribute such as big/little, short/tall, etc.; sort objects into groups by a given attribute.

Cluster	**Describe and Compare Measurable Attributes**
Objectives	**Students will**
M.K.MD.1	describe measurable attributes of objects, such as length or weight and describe several measurable attributes of a single object.
M.K.MD.2	directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.
Cluster	**Classify Objects and Count the Number of Objects in Each Category**
Objectives	**Students will**
M.K.MD.3	classify objects into given categories, count the numbers of objects in each category, and sort the categories by count. Category counts should be limited to less than or equal to 10.

Grade K	**Mathematics**			
Standard	Geometry			
Performance Descriptors M.PD.K.G				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Kindergarten students at the distinguished level in	Kindergarten students at the above mastery level in	Kindergarten students at the mastery level in	Kindergarten students at the partial mastery level in	Kindergarten students at the novice level in mathematics:

mathematics: rearrange, draw a representation and describe shapes and their attributes; justify the likenesses and differences of two- and three-dimensional shapes.	mathematics: create and design with shapes and describe attributes; relate two- and three-dimensional shapes to shapes within the environment.	mathematics: identify, name and describe two- and three-dimensional shapes in the environment, by their orientation, size and relative positions; analyze, compare and describe two- and three-dimensional shapes; model, build and draw shapes, and use simple shapes to compose larger ones.	mathematics: match, name and describe some two and three dimensional shapes; sort two- and three-dimensional shapes.	match and know the names of some shapes in the environment; match two- and three-dimensional shapes.

Cluster	**Identify and Describe Shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders and spheres)**
Objectives	**Students will**
M.K.G.1	describe objects in the environment using names of shapes and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind* and *next to*.
M.K.G.2	correctly name shapes regardless of their orientations or overall size.
M.K.G.3	identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").
Cluster	**Analyze, Compare, Create and Compose Shapes**
Objectives	**Students will**
M.K.G.4	analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).
M.K.G.5	model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
M.K.G.6	compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*

Mathematics - Grade 1

In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of and composing and decomposing geometric shapes.

1. Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart and compare situations to develop meaning for the operations of addition and subtraction and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

2. Students develop, discuss and use efficient, accurate and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

3. Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.[1]

4. Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

[1] Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 1	**Mathematics**
Standard	Operations and Algebraic Thinking
Performance Descriptors M.PD.1.OA	

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
First grade students at the distinguished level in mathematics:	First grade students at the above mastery level in mathematics:	First grade students at the mastery level in mathematics:	First grade students at the partial mastery level in mathematics:	First grade students at the novice level in mathematics:
represent and justify solutions to addition and subtraction problems;	communicate solutions to addition and subtraction problems to others;	analyze relationships between quantities in problem solving situations, use tools to represent the problem and determine the solution;	with tools and assistance, plan solution pathway for solving addition and subtraction problems;	given appropriate number of objects, model adding and subtracting quantities to 20;
compare and justify the effectiveness of different properties of operations in solving the same equation;	explain the use of different properties of operations to others;	flexibly apply different properties of operations to find sums and differences;	recognize different properties of operations;	use objects to model properties of operations;
explain and justify strategies to calculate fluently to 20;	use strategies to calculate fluently to 20;	use strategies to calculate accurately to 20 and fluently to 10;	use strategies to calculate accurately to10;	calculate fluently to 5;
construct and respond to arguments about equivalency in number sentences.	communicate conclusions regarding the relationship of the equal sign to the accuracy of a number sentence.	use the equal sign consistently and appropriately in number sentences and determine an unknown quantity.	model both sides of a number sentence to determine if number sentence is true.	recognize the equal sign.

Cluster	**Represent and Solve Problems Involving Addition and Subtraction**
Objectives	**Students will**
M.1.OA.1	use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions, e.g., by using objects, drawings and equations with a symbol for the unknown number to represent the problem.
M.1.OA.2	solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings and equations with a symbol for the unknown number to represent the problem.
Cluster	**Understand and Apply Properties of Operations and the Relationship between Addition and Subtraction**
Objectives	**Students will**
M.1.OA.3	apply properties of operations as strategies to add and subtract. *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative Property of Addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 +*

	10 = 12. (Associative Property of Addition.) (Students need not use formal terms for these properties.)
M.1.OA.4	understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*
Cluster	**Add and Subtract within 20**
Objectives	**Students will**
M.1.OA.5	relate counting to addition and subtraction (e.g., by counting on 2 to add 2).
M.1.OA.6	add and subtract within 20, demonstrating fluency for addition and subtraction within 10 and use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8= 4); and creating equivalent but easier or known sums (e.g., adding 6 +7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).
Cluster	**Work with Addition and Subtraction Equations**
Objectives	**Students will**
M.1.OA.7	understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1,5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*
M.1.OA.8	determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 +? = 11, 5 = □ – 3, 6 + 6 = □.*

Grade 1	**Mathematics**			
Standard	Numbers and Operations in Base Ten			
Performance Descriptors M.PD.1.NBT				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
First grade students at the distinguished level in mathematics:	First grade students at the above mastery level in mathematics:	First grade students at the mastery level in mathematics:	First grade students at the partial mastery level in mathematics:	First grade students at the novice level in mathematics:
represent and explain various counting patterns beyond 120;	use various number patterns to count within 120;	count, read, represent and write numerals within 120;	read, represent and write numerals within 99;	count and represent numerals within 99 and read and write numerals within 20;
analyze and justify the reasoning about the value of each digit;	communicate reasoning of the comparison of two-digit numbers;	make sense of the quantities represented by each digit in any two-digit number and use symbols to express the comparison of any two two-digit numbers;	use groups of objects to compare the magnitude of two two-digit numbers;	represent a two-digit number with objects;
create models to illustrate strategies.	justify strategies to solve problems involving place	use and explain strategies that reflect understanding of	model addition of any single-digit number to any	add ten to any single-digit number.

	value.	place value and properties of operations to add within 100 and subtract multiples of 10 from within 100 fluently.	multiple of 10 within 100 and subtract 10 from any two-digit number.	

Cluster	**Extend the Counting Sequence**
Objectives	**Students will**
M.1.NBT.1	count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.
Cluster	**Understand Place Value**
Objectives	**Students will**
M.1.NBT.2	understand the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones — called a "ten." b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight or nine tens (and 0 ones).
M.1.NBT.3	compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.
Cluster	**Use Place Value Understanding and Properties of Operations to Add and Subtract**
Objectives	**Students will**
M.1.NBT.4	add within 100, including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used and understand that in adding two-digit numbers, one adds tens and tens, ones and ones and sometimes it is necessary to compose a ten.
M.1.NBT.5	given a two-digit number, mentally find 10 more or 10 less than the number, without having to count and explain the reasoning used.
M.1.NBT.6	subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences) using concrete models or drawings and strategies based on place value, properties of operations and/or the relationship between addition and subtraction and relate the strategy to a written method and explain the reasoning used.

Grade 1	**Mathematics**			
Standard	Measurement and Data			
Performance Descriptors M.PD.1.MD				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
First grade students at the distinguished level in mathematics: demonstrate and justify the	First grade students at the above mastery level in mathematics: use various units of	First grade students at the mastery level in mathematics: compare and order three	First grade students at the partial mastery level in mathematics: compare and order three	First grade students at the novice level in mathematics: compare two objects by

need for a standard unit of measure and, make sound decisions about which unit of measure is appropriate; flexibly use time in everyday life; determine multiple ways to represent and interpret the same data.	measure to analyze the relationship between a measurement and the tool used; make connections between time and events in everyday life; design a method of collecting data.	objects according to length measured accurately by the repeated use of a nonstandard unit; tell and write time using analog and digital clocks to the hour and half-hour; organize, represent and interpret data.	objects by length; tell time on the hour using an analog clock; read and interpret data with up to three categories.	length; tell time on the hour using a digital clock; read displayed data.

Cluster	**Measure Lengths Indirectly and by Iterating Length Units**
Objectives	**Students will**
M.1.MD.1	order three objects by length and compare the lengths of two objects indirectly by using a third object.
M.1.MD.2	express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end and understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*
Cluster	**Tell and Write Time**
Objectives	**Students will**
M.1.MD.3	tell and write time in hours and half-hours using analog and digital clocks.
Cluster	**Represent and Interpret Data**
Objectives	**Students will**
M.1.MD.4	organize, represent, interpret data with up to three categories, ask and answer questions about the total number of data points, how many in each category and how many more or less are in one category than in another.

Grade 1	**Mathematics**
Standard	Geometry

Performance Descriptors M.PD.1.G

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
First grade students at the distinguished level in mathematics: use nets (paper templates) to construct three-dimensional shapes and partition two-dimensional	First grade students at the above mastery level in mathematics: communicate with a degree of precision the attributes of composite shapes to be built by others and partition	First grade students at the mastery level in mathematics: build and draw shapes with defining attributes, build new shapes from composite shapes, partition circles and	First grade students at the partial mastery level in mathematics: distinguish defining attributes of two- and three-dimensional shapes and identify halves and fourths	First grade students at the novice level in mathematics: identify attributes of two-dimensional shapes, use two- and three- dimensional shapes to create composite

shapes into sixths and eighths.	circles and rectangles into thirds.	rectangles into halves and fourths and use appropriate vocabulary to describe the results.	of circles and rectangles.	shapes and identify halves of circles and rectangles.

Cluster	**Reason with Shapes and Their Attributes**
Objectives	**Students will**
M.1.G.1	distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size), build and draw shapes to possess defining attributes.
M.1.G.2	compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones and right circular cylinders) to create a composite shape and compose new shapes from the composite shape. (Students do not need to learn formal names such as "right rectangular prism.")
M.1.G.3	partition circles and rectangles into two and four equal shares, describe the shares using the words *halves, fourths* and *quarters* and use the phrases *half of, fourth of* and *quarter of*, describe the whole as two of, or four of the shares and understand for these examples that decomposing into more equal shares creates smaller shares.

Mathematics - Grade 2

In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

1. Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens and multiples of hundreds, tens and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

2. Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

3. Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

4. Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 2	**Mathematics**			
Standard	Operations and Algebraic Thinking			
Performance Descriptors M.PD.2.OA				
Distinguished	Above Mastery	Distinguished	Above Mastery	Distinguished

Second grade students at the distinguished level in mathematics:	Second grade students at the above mastery level in mathematics:	Second grade students at the mastery level in mathematics:	Second grade students at the partial mastery level in mathematics:	Second grade students at the novice level in mathematics:
analyze givens, constraints, relationships, and goals to plan a solution pathway and justify answers;	Check answers to problems using a different method and assurance answers are reasonable within the context of the problem;	make sense of quantities, relationships in problem situations, and represent symbolically to solve problem;	solve problems with unknowns in various positions;	solve one-step word problems by using drawing and concrete materials;
communicate carefully formulated explanations of mental math strategies;	flexibly use mental math strategies	use various mental strategies and make use of patterns and structures to fluently compute sums and differences;	use mental strategies to add and subtract within 20;	add and subtract within 20 using models;
recognize sums could be expressed as n groups of x objects.	flexibly work with groups of objects representing the sum as repeated addition of equal addends in multiple ways.	represent the sum of a group of objects as the repeated addition of equal addends.	write an addition equation to represent the number of objects in a group.	determine the total number of objects in a group and whether that number is odd or even.

Cluster	**Represent and Solve Problems Involving Addition and Subtraction**
Objectives	**Students will**
M.2.OA.1	use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions, e.g. by using drawings and equations with a symbol for the unknown number to represent the problem.
Cluster	**Add and Subtract within 20**
Objectives	**Students will**
M.2.OA.2	fluently add and subtract within 20 using mental strategies and by end of Grade 2, know from memory all sums of two one-digit numbers.
Cluster	**Work with Equal Groups of Objects to Gain Foundations for Multiplication**
Objectives	**Students will**
M.2.OA.3	determine whether a group of objects (up to 20) has an odd or even number of members, e.g. by pairing objects or counting them by 2s and write an equation to express an even number as a sum of two equal addends.
M.2.OA.4	use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns and write an equation to express the total as a sum of equal addends.

Grade 2	**Mathematics**
Standard	Number and Operations in Base Ten
Performance Descriptors M.PD.2.NBT	

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Second grade students at the distinguished level in mathematics: communicate precisely and justify to others the meaning of the symbols used in number comparisons with 1000; calculate fluently and give carefully formulated explanations to justify answers.	Second grade students at the above mastery level in mathematics: make sense of quantities within 1000 and explain how the relationships of the comparisons are reasonable; flexibly use properties of operations when adding and subtracting numbers within 1,000 and use different methods to check answers.	Second grade students at the mastery level in mathematics: make sense of quantities within 1000 using place value to make comparisons and represent those relationships symbolically; use strategies based on place value, properties of operations and number relationships to add and subtract numbers within 1000.	Second grade students at the partial mastery level in mathematics: model, read, write and compare numerals within 1000 with or without manipulatives; add and subtract within 1,000 including composing or decomposing tens or hundreds.	Second grade students at the novice level in mathematics: model numbers within 1000 using base-ten blocks or drawings; or manipulatives; add and subtract within 1,000 using concrete models or drawings.

Cluster	**Understand Place Value**
Objectives	**Students will**
M.2.NBT.1	understand that the three digits of a three-digit number represent amounts of hundreds, tens and ones; e.g. 706 equals 7 hundreds, 0 tens and 6 ones and understand the following as special cases: a. 100 can be thought of as a bundle of ten tens – called a "hundred." b. numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two , three, four, five, six, seven, eight or nine hundreds (and 0 tens and 0 ones).
M.2.NBT.2	count within 1000 and skip-count by 5s, 10s and 100s.
M.2.NBT.3	read and write numbers to 1000 using base-ten numerals, number names and expanded form.
M.2.NBT.4	compare two three-digit numbers based on meanings of the hundreds, tens and ones digits, using >, = and < symbols to record the results of comparisons.
Cluster	**Use Place Value Understanding and Properties of Operations to Add and Subtract**
Objectives	**Students will**
M.2.NBT.5	fluently add and subtract within 100 using strategies based on place value, properties of operations and/or the relationship between addition and subtraction.
M.2.NBT.6	add up to four two-digit numbers using strategies based on place value and properties of operations.
M.2.NBT.7	add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations and/or the relationship between addition and subtraction, relate the strategy to a written method and understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones and sometimes it is necessary to compose or decompose tens or hundreds.
M.2.NBT.8	mentally add 10 or 100 to a given number 100-900 and mentally subtract 10 or 100 from a given number 100-900.
M.2.NBT.9	explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawing or objects.)

Grade 2	**Mathematics**			
Standard	Measurement and Data			
Performance Descriptors M.PD.2.MD				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Second grade students at the distinguished level in mathematics:	Second grade students at the above mastery level in mathematics:	Second grade students at the mastery level in mathematics:	Second grade students at the partial level in mathematics:	Second grade students at the novice level in mathematics:
provide carefully formulated explanations to justify measurements;	communicate proper use of measurement tools;	estimate length of objects and select the correct tools to accurately measure and compare length;	select appropriate tools to measure and compare lengths of objects;	given the appropriate tool measure the length of objects;
analyze and justify representations and solutions;	communicate representations and solutions;	solve addition and subtraction word problems within 100 involving length and represent quantities of length using number lines and drawings;	add and subtract to solve word problems involving the length of objects;	use concrete models to solve addition word problems involving length of objects;
tell time to the nearest minute, apply monetary skills to real-world situations and evaluate the reasonableness of their conclusions;	communicate precisely how to accurately tell and write time and create word problems involving money;	accurately tell and write time to the nearest five minutes on analog and digital clocks and solve word problems involving money;	read and write time on an analog clock to the nearest hour and half-hour and count the value of coins;	read and write time to the nearest five minutes on digital clocks and recognize the value of coins;
reason deductively about data showing regularity or trends.	represent and interpret data from various graphs, solve word problems, and communicate important features of data.	collect and represent measurement data up to four categories and solve simple problems through the interpretation of the data presented.	collect measurement data and create multiple representations using at least three categories.	given measurement data, create a simple representation.

Cluster	**Measure and Estimate Lengths In Standard Units**
Objectives	**Students will**
M.2.MD.1	measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks and measuring tapes.
M.2.MD.2	measure the length of an object twice, using length units of different lengths for the two measurements, describe how the two measurements relate to the size of the unit chosen and compare and contrast plane and solid geometric shapes.

M.2.MD.3	estimate lengths using units of inches, feet, centimeters and meters.
M.2.MD.4	measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.
Cluster	**Relate Addition and Subtraction to Length**
Objectives	**Students will**
M.2.MD.5	use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
M.2.MD.6	represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0,1,2,... and represent whole-number sums and differences within 100 on a number line diagram.
Cluster	**Work with Time and Money**
Objectives	**Students will**
M.2.MD.7	tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
M.2.MD.8	solve word problems involving dollar bills, quarters, dimes, nickels and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*
Cluster	**Represent and Interpret Data**
Objectives	**Students will**
M.2.MD.9	generate measurement data by measuring lengths of several objects to the nearest whole unit or by making repeated measurements of the same object and show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
M.2.MD.10	draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories and solve simple put-together, take-apart and compare problems using information presented in a bar graph.

Grade 2	**Mathematics**			
Standard	Geometry			
Performance Descriptors M.PD.2.G				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Second grade students at the distinguished level in mathematics: justify the relationship among shapes and that equal shares of identical wholes do not need to have the same shape.	Second grade students at the above mastery level in mathematics: analyze and communicate various ways to partition identical shapes into equal shares.	Second grade students at the mastery level in mathematics: identify and draw shapes given attributes, partition rectangles using congruent squares and count the number of squares, use appropriate vocabulary while partitioning shapes into equal pieces and recognize equal shares may	Second grade students at the partial mastery level in mathematics: identify shapes, name their attributes and recognize equal shares of shapes.	Second grade students at the novice level in mathematics: recognize basic shapes and equal shares of rectangles and circles and given a partitioned rectangle count the number of squares.

		have different shapes.		

Cluster	**Reason with Shapes and Their Attributes**
Objectives	**Students will**
M.2.G.1	recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces (sizes are compared directly or visually, not compared by measuring) and identify triangles, quadrilaterals, pentagons, hexagons and cubes.
M.2.G.2	partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
M.2.G.3	partition circles and rectangles into two, three or four equal shares, describe the shares using the words *halves, thirds, half of, a third of,* etc., describe the whole as two halves, three thirds, four fourths and recognize that equal shares of identical wholes need not have the same shape.

Mathematics - Grade 3

In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

1. Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

2. Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

3. Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication and justify using multiplication to determine the area of a rectangle.

4. Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 3	**Mathematics**
Standard	Operations and Algebraic Thinking
Performance Descriptors M.PD.3.OA	

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Third grade students at the distinguished level in mathematics:	Third grade students at the above mastery level in mathematics:	Third grade students at the mastery level in mathematics:	Third grade students at the partial mastery level in mathematics:	Third grade students at the novice level in mathematics:
justify reasoning when using multiplication, division, and the placement of unknowns when solving real-world problems;	create problems demonstrating understanding of multiplication and division;	interpret products and whole-number quotients of whole numbers and solve problems involving unknowns using multiplication and division;	solve problems involving multiplication and division using manipulatives;	build and separate arrays with counters to represent multiplication and division facts;
communicate solutions to real-world problems clarifying the relationship between multiplication and division;	use properties of operations to plan pathways to solve problems;	apply properties of operations to solve multiplication, unknown factor, and division problems;	use various properties of operations to solve multiplication and division problems;	use commutative property of multiplication to solve problems involving basic multiplication facts;
develop and share strategies for recalling and relating multiplication and division facts;	explain strategies for recalling multiplication and division facts;	use strategies to fluently multiply and divide within 100;	use strategies to multiply two one-digit numbers;	use tools to multiply within 100;
analyze connections among operations and patterns when modeling and solving real-world problems.	plan and communicate a pathway for solving problems and justify solutions.	use the four operations and arithmetic patterns to solve two-step word problems and use estimation to check reasonableness of answers.	solve problems using the four operations and arithmetic patterns.	solve basic problems involving the four operations using tools.

Cluster	**Represent and Solve Problems Involving Multiplication and Division**
Objectives	**Students will**
M.3.OA.1	interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe context in which a total number of objects can be expressed as 5 × 7.*
M.3.OA.2	interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*
M.3.OA.3	use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
M.3.OA.4	determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8× ? = 48, 5 =□÷ 3, 6 × 6 = ?.*
Cluster	**Understand Properties of Multiplication and the Relationship between Multiplication and Division**

Objectives	Students will
M.3.OA.5	apply properties of operations as strategies to multiply and divide.*(Students need not use formal terms for these properties.) Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*
M.3.OA.6	understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*
Cluster	**Multiply and Divide within 100**
Objectives	**Students will**
M.3.OA.7	fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations and by the end of Grade 3, know from memory all products of two one-digit numbers.
Cluster	**Solve Problems Involving the Four Operations and Identify and Explain Patterns in Arithmetic**
Objectives	**Students will**
M.3.OA.8	solve two-step word problems using the four operations, represent these problems using equations with a letter standing for the unknown quantity and assess the reasonableness of answers using mental computation and estimation strategies including rounding. (*This standard is limited to problems posed with whole numbers and having whole number answers; students should know how to perform operations in the conventional order when there are no parenthesis to specify a particular order (Order of Operations).)*
M.3.OA.9	identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. *For example, observe that 4 times a number is always even and explain why 4 times a number can be decomposed into two equal addends.*

Grade 3	**Mathematics**			
Standard	Numbers & Operations in Base Ten			
Performance Descriptors M.PD.3.NBT				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Third grade students at the distinguished level in Mathematics: communicate understanding of place value, multiples and properties of operations to justify solutions to real-life problems.	Third grade students at the above mastery level in Mathematics: justify the use of rounding, multiples and the relationship of arithmetic operations when solving real-life problems.	Third grade students at the mastery level in Mathematics: apply understanding of place value when rounding whole numbers, relate addition and subtraction using properties of operations and multiply one-digit numbers by multiples of ten.	Third grade students at the mastery level in Mathematics: make sense of place value to add, subtract, round or find multiples using tools such as number line or 100 board.	Third grade students at the novice level in Mathematics: use models to add or subtract.

Cluster	**Use Place Value Understanding and Properties of Operations to Perform Multi-digit Arithmetic.** *(A range of algorithms may be used.)*
Objectives	**Students will**
M.3.NBT.1	use place value understanding to round whole numbers to the nearest 10 or 100.
M.3.NBT.2	fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations and/or the relationship between addition and subtraction.
M.3.NBT.3	multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Grade 3	**Mathematics**			
Standard	Number and Operations – Fractions (*Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)*			
Performance Descriptors M.PD.3.NF				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Third grade students at the distinguished level in mathematics: justify and communicate the use of fractions to solve real-world problems.	Third grade students at the above mastery level in mathematics: use models or a number line to explain comparisons of fractions and check reasonableness of the comparisons.	Third grade students at the mastery level in mathematics: make sense of fractions as equal parts of a whole or points on a number line, explain equivalent fractions and compare fractions using various criteria and symbols.	Third grade students at the partial mastery level in mathematics: use various models or a number line to compare fractions.	Third grade students at the novice level in mathematics: identify fractions using models.

Cluster	**Develop Understanding of Fractions as Numbers**
Objectives	**Students will**
M.3.NF.1	understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts and understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.
M.3.NF.2	understand a fraction as a number on the number line and represent fractions on a number line diagram a. represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts and recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. b. represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0 and recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.
M.3.NF.3	explain equivalence of fractions in special cases and compare fractions by reasoning about their size a. understand two fractions as equivalent (equal) if they are the same size or the same point on a number line, b. recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3) and explain why the fractions are equivalent, e.g., by using a visual fraction model, c. express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers (*Examples: Express 3 in*

	the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.), d. compare two fractions with the same numerator or the same denominator by reasoning about their size, recognize that comparisons are valid only when the two fractions refer to the same whole, record the results of comparisons with the symbols >, = or < and justify the conclusions, e.g., by using a visual fraction model.

Grade 3	**Mathematics**			
Standard	Measurement and Data			
Performance Descriptors M.PD.3.MD				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Third grade students at the distinguished level in mathematics:	Third grade students at the above mastery level in mathematics:	Third grade students at the mastery level in mathematics:	Third grade students at the partial mastery level in mathematics:	Third grade students at the novice level in mathematics:
research and appraise real-world examples of situations by measuring and estimating time, liquid volume or masses of objects;	analyze real-world problems by using appropriate tools to measure and estimate time, liquid volume and masses of objects;	solve problems using measuring and estimating of liquid volume, object mass, and intervals of time to the nearest minute;	tell time to the nearest five minute interval and choose appropriate tools to measure liquid volume and masses of objects;	tell time in fifteen minute intervals; measure liquid volume and masses of objects;
organize and present real world data on a variety of graphs to justify solutions to problems based on data;	check for reasonableness of solutions to problems using data from graphs and line plots;	create and use graphs to solve one- and two- step problems comparing data; measure objects to the nearest ½ or ¼ inch and create a line plot;	use data from graphs and line plots to solve one-step problems;	find and compare data on graphs and line plots;
communicate the use of area in the real-world and justify short cuts for finding area using addition, multiplication or tiling of non-rectilinear shapes;	critique the correspondences between different approaches in finding area;	explain how area is determined in more than one way and area's relationship to addition and multiplication;	find the area of various shapes by counting unit squares and given the algorithm for multiplication, multiply to find area of rectangles;	count unit squares to find area of rectilinear shapes;
construct viable arguments and critique the reasoning of others in solving real-world problems involving perimeter and areas.	communicate ways to compare and contrast perimeters and areas of real-world plane figures.	find perimeters of real-world plane figures specifying linear units and create rectangles with the same area and different	find perimeter of plane figures without all sides labeled.	find perimeter when given all side lengths.

		perimeters and vice versa.		

Cluster	**Solve Problems Involving Measurement and Estimation of Intervals of Time, Liquid Volumes and Masses of Objects**
Objectives	**Students will**
M.3.MD.1	tell and write time to the nearest minute, measure time intervals in minutes and solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
M.3.MD.2	measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg) and liters (*l*) (*Excludes compound units such as* cm^3 *and finding the geometric volume of a container.*) and subtract, multiply or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (*Excludes multiplicative comparison problems - problems involving notions of "times as much".*)
Cluster	**Represent and Interpret Data**
Objectives	**Students will**
M.3.MD.3	draw a scaled picture graph and a scaled bar graph to represent a data set with several categories and solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*
M.3.MD.4	generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch and show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves or quarters.
Cluster	**Geometric Measurement: Understand Concepts of Area and Relate Area to Multiplication and to Addition**
Objectives	**Students will**
M.3.MD.5	recognize area as an attribute of plane figures and understand concepts of area measurement a. a square with side length 1 unit, called "a unit square," is said to have "one square unit" of area and can be used to measure area, b. a plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.
M.3.MD.6	measure areas by counting unit squares (square cm, square m, square in, square ft and improvised units).
M.3.MD.7	relate area to the operations of multiplication and addition a. find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths, b. multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems and represent whole-number products as rectangular areas in mathematical reasoning, c. use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* × *b* and *a* × *c and u*se area models to represent the distributive property in mathematical reasoning, d. recognize area as additive and find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.
Cluster	**Geometric Measurement: Recognize Perimeter as an Attribute of Plane Figures and Distinguish between Linear and Area Measures**
Objectives	**Students will**
M.3.MD.8	solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Grade 3	**Mathematics**			
Standard	Geometry			
Performance Descriptors M.PD.3.G				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Third grade students at the distinguished level in mathematics: analyze the relationship among shapes sharing attributes, justify the placement of shapes into various groups, and demonstrate equal parts of shapes as fractions of the whole.	Third grade students at the above mastery level in mathematics: produce a display comparing and contrasting shapes by attributes and identify equal parts of shapes as fractions of the shape.	Third grade students at the mastery level in mathematics: classify and describe shapes by attributes showing that some groups overlap; model equal parts of various shapes to express the part as a fraction of the whole.	Third grade students at the partial mastery level in mathematics: classify shapes by one attribute and show fractional parts.	Third grade students at the novice level in mathematics: identify quadrilaterals and sort concrete shapes by number of sides.

Cluster	**Reason with Shapes and Their Attributes**
Objectives	**Students will**
M.3.G.1	understand that shapes in different categories (e.g., rhombuses, rectangles and others) may share attributes (e.g., having four sides), that the shared attributes can define a larger category (e.g. quadrilaterals), recognize rhombuses, rectangles and squares as examples of quadrilaterals and draw examples of quadrilaterals that do not belong to any of these subcategories.
M.3.G.2	partition shapes into parts with equal areas and express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as ¼ or the area of the shape.

Mathematics - Grade 4

In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures and symmetry.

1. Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value and properties of operations, in particular the distributive property, as they develop, discuss and use efficient, accurate and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations and the relationship of division to multiplication as they develop, discuss and use efficient, accurate and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients and interpret remainders based upon the context.

2. Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

3. Students describe, analyze, compare and classify two-dimensional shapes. Through building, drawing and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 4	**Mathematics**			
Standard	Operations and Algebraic Thinking			
Performance Descriptors M.PD.4.OA				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Fourth grade students at the	Fourth grade students at the	Fourth grade students at the	Fourth grade students at the	Fourth grade students at the

distinguished level in mathematics:	above mastery level in mathematics:	mastery level in mathematics:	partial mastery level in mathematics:	novice level in mathematics:
justify, communicate, defend conclusions, and respond to the arguments of others;	use various strategies to solve and check multi-step word problems and precisely communicate procedures used;	distinguish between multiplicative and additive reasoning, apply the four operations with whole numbers to solve multi-step word problems, represent problems with equations containing unknowns, and evaluate the reasonableness of the results;	distinguish between multiplicative and additive reasoning and apply the four operations with whole numbers to solve multi-step word problems;	apply the four operations with whole numbers to solve simple one-step word problems;
analyze and reflect on relationships among factors and multiples and draw conclusions;	identify significance of the factor/multiple relationships;	find and make connections between factors/multiples and prime/composite numbers;	given a list of multiples of a number, determine the number and given a number, determine some of its multiples and factors;	state multiplication facts and use concrete objects or pictures to model factors and multiples;
analyze relationships in a given pattern and discover a rule.	- justify and communicate a pattern or given rule.	generate and/or discern a pattern or structure when given a rule.	extend or complete a pattern or structure.	identify missing elements in a given pattern.

Cluster	**Use the Four Operations with Whole Numbers to Solve Problems**
Objectives	**Students will**
M.4.OA.1	interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5 and represent verbal statements of multiplicative comparisons as multiplication equations.
M.4.OA.2	multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem and distinguishing multiplicative comparison from additive comparison.
M.4.OA.3	solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted, represent these problems using equations with a letter standing for the unknown quantity and assess the reasonableness of answers using mental computation and estimation strategies including rounding.
Cluster	**Gain Familiarity with Factors and Multiples**
Objectives	**Students will**
M.4.OA.4	find all factor pairs for a whole number in the range 1–100, recognize that a whole number is a multiple of each of its factors, determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number and determine whether a given whole number in the range 1–100 is prime or composite.
Cluster	**Generate and Analyze**
Objectives	**Students will**

M.4OA.5	generate a number or shape pattern that follows a given rule and identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Grade 4	**Mathematics**
Standard	Number and Operations in Base Ten

Performance Descriptors M.PD.4.NBT

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Fourth grade students at the distinguished level in mathematics: connect place value to other mathematical concepts, including time, money, and measurement; communicate reasoning, analyze situations, and justify solutions.	Fourth grade students at the above mastery level in mathematics: explain place value relationships and knowledge of rounding and justify reasoning; evaluate the reasonableness of intermediate results while performing multi-digit arithmetic.	Fourth grade students at the mastery level in mathematics: demonstrate understanding of place value and rounding of whole numbers; illustrate and explain place value and apply properties of operations to perform multi-digit arithmetic.	Fourth grade students at the partial mastery level in mathematics: make sense of place value relationships for multi-digit whole numbers; apply place value understanding and properties of operations to perform simple multi-digit arithmetic.	Fourth grade students at the novice level in mathematics: use concrete objects or pictures to help conceptualize and understand the base ten system; procedurally perform addition and subtraction problems and know basic multiplication facts.

Cluster	**Generalize Place Value Understanding for Multi-digit Whole Numbers**
Objectives	**Students will**
M.4.NBT.1	recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*
M.4.NBT.2	read and write multi-digit whole numbers using base-ten numerals, number names and expanded form and compare two multi-digit numbers based on meanings of the digits in each place, using >, = and < symbols to record the results of comparisons.
M.4.NBT.3	use place value understanding to round multi-digit whole numbers to any place.
Cluster	**Use Place Value Understanding and Properties of Operations to Perform Multi-digit Arithmetic**
Objectives	**Students will**
M.4.NBT.4	fluently add and subtract multi-digit whole numbers using the standard algorithm.
M.4.NBT.5	multiply a whole number of up to four digits by a one-digit whole number, multiply two two-digit numbers, using strategies based on place value and the properties of operations and illustrate and explain the calculation by using equations, rectangular arrays and/or area models.
M.4.NBT.6	find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations and/or the relationship between multiplication and division and illustrate and explain the calculation

	by using equations, rectangular arrays and/or area models.

Grade 4	**Mathematics**			
Standard	**Number and Operations - Fractions**			
Performance Descriptors M.PD.4.NF				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Fourth grade students at the distinguished level in mathematics: use and apply knowledge of equivalent fractions as a strategy to solve real-world problems in new situations; extend knowledge of fractions by applying mathematics to solve problems arising in everyday life, society and the workplace; justify correspondences between fraction and decimal comparisons.	Fourth grade students at the above mastery level in mathematics: communicate understanding of fraction equivalence and ordering to others and justify reasoning; analyze and explain fraction relationships for adding, subtracting, and multiplying; explain decimal to fraction conversions.	Fourth grade students at the mastery level in mathematics: extend understanding of fraction equivalence and ordering; extend understanding of addition, subtraction, and multiplication in whole numbers to fractions; understand and compare decimal notation for fractions.	Fourth grade students at the partial mastery level in mathematics: make sense of fraction equivalence and ordering using concrete objects; extend knowledge of adding and subtracting to fractions; use concrete objects or pictures to compare fractions to decimals.	Fourth grade students at the novice level in mathematics: recognize fractions as numbers using concrete objects, as necessary; use concrete objects or pictures to help conceptualize knowledge of fractions; use concrete objects or pictures to illustrate fraction to decimal relationships.

Cluster	**Extend Understanding of Fraction Equivalence and Ordering**
Objectives	**Students will**
M.4.NF.1	explain why a fraction *a*/*b* is equivalent to a fraction ($n \times a$)/($n \times b$) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size and use this principle to recognize and generate equivalent fractions.
M.4.NF.2	compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as ½, recognize that comparisons are valid only when the two fractions refer to the same whole and record the results of comparisons with symbols >, = or <, and justify the conclusions, e.g., by using a visual fraction model.
Cluster	**Build Fractions from Unit Fractions by Applying and Extending Previous Understandings of Operations on Whole Numbers**
Objectives	**Students will**
M.4.NF.3	understand a fraction *a*/*b* with $a > 1$ as a sum of fractions 1/*b* a. understand addition and subtraction of fractions as joining and separating parts referring to the same whole, *b.* decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation and justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 +*

	1/8 + 1/8; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8, c. add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction and/or by using properties of operations and the relationship between addition and subtraction, d. solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
M.4.NF.4	apply and extend previous understandings of multiplication to multiply a fraction by a whole number a. understand a fraction *a*/*b* as a multiple of 1/*b*, (*For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).)* b. understand a multiple of *a*/*b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number, (*For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. In general, n × (a/b) = (n × a)/b.)* c. solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. (*For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?)*
Cluster	**Understand Decimal Notation for Fractions and Compare Decimal Fractions**
Objectives	**Students will**
M.4.NF.5	express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.)
M.4.NF.6	use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*
M.4.NF.7	compare two decimals to hundredths by reasoning about their size, recognize that comparisons are valid only when the two decimals refer to the same whole and record the results of comparisons with the symbols >, = or < and justify the conclusions, e.g., by using a visual model.

Grade 4	**Mathematics**			
Standard	Measurement and Data			
Performance Descriptors M.PD.4.MD				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Fourth grade students at the distinguished level in mathematics: analyze arguments and justify reasoning;	Fourth grade students at the above mastery level in mathematics: analyze relationships among conversions and give carefully formulated explanations of solutions;	Fourth grade students at the mastery level in mathematics: solve real-world problems involving measurements, conversions, formulas, and use of tools;	Fourth grade students at the below mastery level in mathematics: choose appropriate tools when solving measurement problems;	Fourth grade students at the novice level in mathematics: use given tools to solve measurement problems;

identify a real-world problem, design and conduct experiments involving measurements, record and display data, analyze, and communicate results;	communicate data interpretation clearly and concisely;	record, display in a line plot with fractional coefficients, and interpret given data to solve word problems	represent data in the form of a line plot with fractional coefficients;	read data in the form of a line plot with fractional coefficients;
design a project demonstrating knowledge of angle concepts and measurements and present final product.	explain angle relationships, compute and justify solutions.	understand concepts of angle, measure angles and recognize angle measures as additive.	use a protractor to measure angles.	use concrete objects or pictures to make sense of angles.

Cluster	**Solve Problems Involving Measurement and Conversion of Measurements from a Larger Unit to a Smaller Unit**
Objectives	**Students will**
M.4.MD.1	know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec, within a single system of measurement, express measurements in a larger unit in terms of a smaller unit, record measurement equivalents in a two column table, (For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in.) and generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36).
M.4.MD.2	use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects and money, including problems involving simple fractions or decimals and problems that require expressing measurements given in a larger unit in terms of a smaller unit and represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
M.4.MD.3	apply the area and perimeter formulas for rectangles in real world and mathematical problems. (For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.)
Cluster	**Represent and Interpret Data**
Objectives	**Students will**
M.4.MD.4	make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8) and solve problems involving addition and subtraction of fractions by using information presented in line plots (for example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection).
Cluster	**Geometric Measurement: Understand Concepts of Angle and Measure Angles**
Objectives	**Students will**
M.4.MD.5	recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. an angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle and an angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles, b. an angle that turns through n one-degree angles is said to have an angle measure of n degrees.

M.4.MD.6	measure angles in whole-number degrees using a protractor and sketch angles of specified measure.
M.4.MD.7	recognize angle measure as additive, when an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts and solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Grade 4	**Mathematics**
Standard	Geometry

Performance Descriptors M.PD.4.G

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Fourth grade students at the distinguished level in mathematics: justify conclusions and respond to the arguments of others.	Fourth grade students at the above mastery level in mathematics: communicate identifications and classifications of shapes.	Fourth grade students at the mastery level in mathematics: draw and identify lines and angles, classify shapes by properties of lines and angles and recognize line two-dimensional symmetry.	Fourth grade students at the below mastery level in mathematics: use appropriate tools to draw lines, angles and shapes.	Fourth grade students at the novice level in mathematics: use concrete objects or pictures to identify lines, angles and shapes.

Cluster	**Draw and Identify Lines and Angles and Classify Shapes by Properties of Their Lines and Angles**
Objectives	**Students will**
M.4.G.1	draw points, lines, line segments, rays, angles (right, acute, obtuse) and perpendicular and parallel lines and identify these in two-dimensional figures.
M.4.G.2	classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size, recognize right triangles as a category and identify right triangles.
M.4.G.3	recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts, identify line-symmetric figures and draw lines of symmetry.

Mathematics - Grade 5

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

1. Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

2. Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication and division. They apply their understandings of models for decimals, decimal notation and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

3. Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 5	**Mathematics**
Standard	Operations and Algebraic Thinking
Performance Descriptors M.PD.5.OA	

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Fifth grade students at the distinguished level in mathematics: represent simple real-world situations with numerical expressions; create two or more rules and analyze the described patterns.	Fifth grade students at the above mastery level in mathematics: analyze and justify expressions to determine equivalence; extend numerical patterns, make predictions and draw conclusions based on the patterns.	Fifth grade students at the mastery level in mathematics: evaluate expressions, interpret the meaning of more complex expressions without evaluating them and translate verbal phrases into numerical expressions; analyze rules and patterns, the numerical relationship between the terms of those patterns and represent the relationship of those terms on a coordinate graph.	Fifth grade students at the partial mastery level in mathematics: translate simple verbal phrases into numerical expressions; identify the relationship between terms in a given pattern and recognize the relationship between the ordered pairs and the graph of the terms of the pattern.	Fifth grade students at the novice level in mathematics: evaluate given expressions; determine the rule, the input or output for a numerical pattern and write and graph ordered pairs.

Cluster	**Write and Interpret Numerical Expressions**
Objectives	Students will
M.5.OA.1	use parentheses, brackets or braces in numerical expressions and evaluate expressions with these symbols.
M.5.OA.2	write simple expressions that record calculations with numbers and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*
Cluster	**Analyze Patterns and Relationships**
Objectives	Students will
M.5.OA.3	generate two numerical patterns using two given rules, identify apparent relationships between corresponding terms, form ordered pairs consisting of corresponding terms from the two patterns and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0 and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Grade 5	**Mathematics**
Standard	Numbers and Operations in Base Ten
Performance Descriptors M.PD.5.NBT	

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Fifth grade students at the distinguished level in mathematics: justify the underlying structure and patterns of the place value system and extend the use of powers of ten to decimal numbers; justify strategies and procedures used to solve problems involving multi-digit whole numbers and decimals to hundredths.	Fifth grade students at the above mastery level in mathematics: fluently translate whole and decimal numbers from one form to another and communicate the results; utilize multiple strategies to solve problems involving multi-digit whole numbers and decimals to hundredths.	Fifth grade students at the mastery level in mathematics: discern and explain the pattern of place values; fluently perform operations with multi-digit whole numbers and decimals to hundredths.	Fifth grade students at the partial mastery level in mathematics: apply place value skills to problems involving whole numbers and decimals; demonstrate procedural knowledge of operations with whole numbers and decimals to hundredths.	Fifth grade students at the novice level in mathematics: differentiate patterns in the place value system; demonstrate procedural knowledge of operations with whole numbers.

Cluster	**Understand the Place Value System**
Objectives	**Students will**
M.5.NBT.1	recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
M.5.NBT.2	explain patterns in the number of zeros of the product when multiplying a number by powers of 10, explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10 and use whole-number exponents to denote powers of 10.
M.5.NBT.3	read, write and compare decimals to thousandths a. read and write decimals to thousandths using base-ten numerals, number names and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$, b. compare two decimals to thousandths based on meanings of the digits in each place, using >, = and < symbols to record the results of comparisons.
M.5.NBT.4	use place value understanding to round decimals to any place.
Cluster	**Perform Operations with Multi-digit Whole Numbers and with Decimals to Hundredths**
Objectives	**Students will**
M.5.NBT.5	fluently multiply multi-digit whole numbers using the standard algorithm.
M.5.NBT.6	find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations and/or the relationship between multiplication and division, illustrate and explain the calculation by using equations, rectangular arrays and/or area models.
M.5.NBT.7	add, subtract, multiply and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations and/or the relationship between addition and subtraction, relate the strategy to a written method and explain the reasoning used.

Grade 5	**Mathematics**
Standard 3	Number and Operations – Fractions

Performance Descriptors M.PD.5.NF

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Fifth grade students at the distinguished level in mathematics:	Fifth grade students at the above mastery level in mathematics:	Fifth grade students at the mastery level in mathematics:	Fifth grade students at the partial mastery level in mathematics:	Fifth grade students at the novice level in mathematics:
when given a solution, students work backwards to create a problem situation and justify their thinking;	give carefully formulated explanations of the procedural steps used to solve problems;	estimate and solve word problems involving addition and subtraction of fractions and mixed numbers, assess reasonableness of answers;	add and subtract fractions and mixed numbers with like denominators;	add and subtract fractions with like denominators;
create multiple models to solve real-world problems; defend strategies and appropriateness of models chosen.	create and connect a visual model to procedures used to solve problems.	use visual models and equations while solving real-world problems demonstrating the understanding of multiplication and division of various fractional representations.	given a procedure find quotients and products of fractions and whole numbers.	use models to multiply fractions by whole numbers.

Cluster	**Use Equivalent Fractions as a Strategy to Add and Subtract Fractions**
Objectives	**Students will**
M.5.NF.1	add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)
M.5.NF.2	solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem and use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*
Cluster	**Apply and Extend Previous Understandings of Multiplication and Division to Multiply and Divide Fractions**
Objectives	**Students will**
M.5.NF.3	interpret a fraction as division of the numerator by the denominator (*a*/*b* = *a* ÷ *b*) and solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3 and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*
M.5.NF.4	apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction a. interpret the product (*a*/*b*) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of

	operations a × q ÷ b. *For example, use a visual fraction model to show (2/3) × 4 = 8/3 and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* b. find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths and show that the area is the same as would be found by multiplying the side lengths, multiply fractional side lengths to find areas of rectangles and represent fraction products as rectangular areas.
M.5.NF.5	interpret multiplication as scaling (resizing), by: a. comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication, b. explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case), explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number and relating the principle of fraction equivalence *a/b* = (*n*×*a*)/(*n*×*b*) to the effect of multiplying *a/b* by 1.
M.5.NF.6	solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
M.5.NF.7	apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions (Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.) a. interpret division of a unit fraction by a non-zero whole number and compute such quotients. *For example, create a story context for (1/3) ÷ 4 and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.* b. interpret division of a whole number by a unit fraction and compute such quotients. *For example, create a story context for 4 ÷ (1/5) and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* c. solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many1/3-cup servings are in 2 cups of raisins?*

Grade 5	**Mathematics**			
Standard	Measurement and Data			
Performance Descriptors M.PD.5.MD				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Fifth grade students at the distinguished level in mathematics: justify solutions and create representations of real-world problems involving measurement conversions;	Fifth grade students at the above mastery level in mathematics: estimate solutions and assess the reasonableness of answers while solving multi-step measurement	Fifth grade students at the mastery level in mathematics: solve multi-step problems demonstrating the understanding of the relationships among units of	Fifth grade students at the partial mastery level in mathematics: solve one-step problems involving conversion of units within a measurement system;	Fifth grade students at the novice level in mathematics: convert units within a measurement system;

	problems within a given measurement system;	measurement within a given measurement system;		
pose new problems requiring collection and analysis of data;	explain solutions to real-world problems through the display and interpretation of data;	solve real-world problems through the interpretation of data on a line plot;	interpret data displayed on a line plot;	display data on a line plot;
apply and justify multiplicative reasoning to determine possible dimensions of figures with a given volume.	communicate to others how the formula for volume is derived.	demonstrate the understanding of volume concepts through measuring and the application of formulas to calculate an object's volume.	find volume using additive reasoning.	find volume through hands-on experience.

Cluster	**Convert Like Measurement Units within a Given Measurement System**
Objectives	**Students will**
M.5.MD.1	convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m) and use these conversions in solving multi-step, real-world problems.
Cluster	**Represent and Interpret Data**
Objectives	**Students will**
M.5.MD.2	make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8) and use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*
Cluster	**Geometric Measurement: Understand Concepts of Volume and Relate Volume to Multiplication and to Addition**
Objectives	**Students will**
M.5.MD.3	recognize volume as an attribute of solid figures and understand concepts of volume measurement a. a cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume and can be used to measure volume, b. a solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.
M.5.MD.4	measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft and improvised units.
M.5.AD.5	relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume a. find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base and represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. b. apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real-world and mathematical problems. c. recognize volume as additive and find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

Grade 5	**Mathematics**
Standard	Geometry

Performance Descriptors M.PD.5.G

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Fifth grade students at the distinguished level in mathematics: justify solutions to problems involving points on a coordinate plane; create multiple examples and counterexamples of two-dimensional figures when given a set of attributes and justify thinking.	Fifth grade students at the above mastery level in mathematics: create problems involving points on a coordinate plane; select examples and counterexamples of two-dimensional figures based on a given a set of attributes.	Fifth grade students at the mastery level in mathematics: represent problems on a coordinate plane and find solutions; use precise language to describe, classify and identify relationships among two-dimensional figures based on attributes.	Fifth grade students at the partial mastery level in mathematics: given coordinates plot points in Quadrant I; describe and classify shape based on dimensional attributes.	Fifth grade students at the novice level in mathematics: state the coordinates of a given point; sort two-dimensional figures based on attributes.

Cluster	**Graph Points on the Coordinate Plane to Solve Real-world and Mathematical Problems**
Objectives	Students will
M.5.G.1	use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates and understand that the first number indicates how far to travel from the origin in the direction of one axis and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).
M.5.G.2	represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation.
Cluster	**Classify Two-dimensional Figures into Categories Based on Their Properties**
Objectives	Students will
M.5.G.3	understand that attributes belonging to a category of two dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*
M.5.G.4	classify two-dimensional figures in a hierarchy based on properties.

Mathematics - Grade 6

In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting and using expressions and equations; and (4) developing understanding of statistical thinking.

1. Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

2. Students use the meaning of fractions, the meanings of multiplication and division and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

3. Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

4. Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps and symmetry, considering the context in which the data were collected.

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces and relating the shapes to rectangles. Using these methods, students discuss, develop and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 6	**Mathematics**			
Standard	**Ratios and Proportional Relationships**			
Performance Descriptors M.PD.6.RP				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Sixth grade students at the distinguished level in mathematics: analyze ratios and the relationship to fractions and communicate similarities and differences.	Sixth grade students at the above mastery level in mathematics: create and model problems requiring ratio or proportional reasoning.	Sixth grade students at the mastery level in mathematics: state the meaning of ratio concepts, use ratio reasoning and rates to solve problems.	Sixth grade students at the partial mastery level in mathematics: write ratios describing a relationship between two quantities and make tables of equivalent ratios.	Sixth grade students at the novice level in mathematics: recognize and utilize ratios.

Cluster	**Understand ratio concepts and use ratio reasoning to solve problems.**
Objectives	**Students will**
M.6.RP.1	understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."
M.6.RP.2	understand the concept of a unit rate *a/b* associated with a ratio *a:b* with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." (Expectations for unit rates in this grade are limited to non-complex fractions.)*
M.6.RP.3	use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. make tables of equivalent ratios relating quantities with whole number *b.* measurements, find missing values in the tables and plot the pairs of values on the coordinate plane. Use tables to compare ratios. c. solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* d. find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. e. use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing

	quantities.

Grade 6	**Mathematics**
Standard	**The Number System**

Performance Descriptors M.PD.6.NS

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Sixth grade students at the distinguished level in mathematics:	Sixth grade students at the above mastery level in mathematics:	Sixth grade students at the mastery level in mathematics:	Sixth grade students at the partial mastery level in mathematics:	Sixth grade students at the novice level in mathematics:
communicate understanding of the connections among problems, models and numerical solutions created;	create and model word problems requiring division of fractions for a specific meaning;	model and solve word problems requiring division of fractions by fractions and interpret the quotient in the context of the situation;	solve word problems which require the division of fractions;	find the quotient of fractions;
communicate understanding of the connection between the greatest common factor and the distributive property;	assess reasonableness of computations and give carefully formulated explanations about the difference between greatest common factor and least common multiple;	perform all operations (including the distributive property) fluently with decimals and whole numbers, identify least common multiple and greatest common factor;	determine and identify the difference between common multiples and common factors of numbers;	perform operations with decimals, fractions and whole numbers and identify factors and multiples;
reason inductively that linear relationships exist and communicate how graphs can be used to model problems.	make conjectures about graphs while solving real-world mathematical problems.	make sense of quantities and relationships among rational numbers and absolute values and use graphs to solve real-world problems and discern patterns.	determine the absolute value of rational numbers.	place negative numbers on a number line and offer examples of negative numbers in the real-world.

Cluster	**Apply and extend previous understandings of multiplication and division to divide fractions by fractions.**
Objectives	Students will
M.6.NS.1	interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3.* *(In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area ½ square mi?*
Cluster	**Compute fluently with multi-digit numbers and find common factors and multiples.**

Objectives	Students will
M.6.NS.2	fluently divide multi-digit numbers using the standard algorithm.
M.6.NS.3	fluently add, subtract, multiply and divide multi-digit decimals using the standard algorithm for each operation.
M.6.NS.4	find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*
Cluster	**Apply and extend previous understandings of numbers to the system of rational numbers.**
Objectives	Students will
M.6.NS.5	understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
M.6.NS.6	understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite. b. understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
M.6.NS.7	understand ordering and absolute value of rational numbers. *a.* interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.* *b.* write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write -3^{o} C > -7^{o} C to express the fact that -3^{o} C is warmer than -7^{o} C.* c. understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write $\|-30\| = 30$ to describe the size of the debt in dollars.* d. distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*
M.6.NS.8	solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Grade 6	**Mathematics**			
Standard	**Expressions and Equations**			
Performance Descriptors M.PD.6.EE				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Sixth grade students at the distinguished level in	Sixth grade students at the above mastery level in	Sixth grade students at the mastery level in	Sixth grade students at the partial mastery level in	Sixth grade students at the novice level in mathematics:

mathematics: create real-life representations of given algebraic expressions;	mathematics: translate algebraic expressions into words and state the meanings of symbols;	mathematics: extend reasoning from numerical to algebraic expressions, identify and simplify equivalent expressions and communicate meaning using appropriate mathematical vocabulary;	mathematics: identify terms in a given algebraic expression;	evaluate algebraic expressions when given a value for the variable;
identify and communicate constraints on variables based on the context of the equation or inequality;	create and identify pathways to solve real-world problems requiring algebraic expressions or inequalities;	use algebraic equations and inequalities to solve real-world problems and understand domains and meanings of variables in different contexts;	evaluate algebraic expressions and inequalities using substitution;	evaluate algebraic equations using number lines;
identify a real-world problem which models using dependent and independent variables, collect and analyze data and find solutions.	create and identify pathways to solve real-world problems involving dependent and independent variables.	analyze relationships between dependent and independent variables, state the meaning of variables, write applicable equations, and analyze using graphs and tables.	identify dependent and independent variables in context.	recognize when a word problem requires two variables.

Cluster	**Apply and extend previous understandings of arithmetic to algebraic expressions.**
Objectives	**Students will**
M.6.EE.1	write and evaluate numerical expressions involving whole-number exponents.
M.6.EE.2	write, read and evaluate expressions in which letters stand for numbers. a. write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.* b. identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* c. evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas V = s3 and A = 6 s2 to find the volume and surface area of a cube with sides of length s = 1/2.*
M.6.EE.3	Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the*

	distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.
M.6.EE.4	identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*
Cluster	**Reason about and solve one-variable equations and inequalities.**
Objectives	Students will
M.6.EE.5	understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
M.6.EE.6	use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number or depending on the purpose at hand, any number in a specified set.
M.6.EE.7	solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.
M.6.EE.8	write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
Cluster	**Represent and analyze quantitative relationships between dependent and independent variables.**
Objectives	Students will
M.6.EE.9	use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.*

Grade 6	**Mathematics**			
Standard	**Geometry**			
Performance Descriptors M.PD.6.G				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Sixth grade students at the distinguished level in mathematics: construct and compare possible models of a three-dimensional figure with a given volume.	Sixth grade students at the above mastery level in mathematics: make and test a conjecture and communicate how changing one dimension affects volume.	Sixth grade students at the mastery level in mathematics: create representations of three-dimensional geometric figures while solving real-world and mathematical problems involving surface area and	Sixth grade students at the partial mastery level in mathematics: using three-dimensional figures and given formulas, determine volume of the figures and area of any face.	Sixth grade students at the novice level in mathematics: identify faces of three-dimensional figures as two-dimensional geometric shapes.

		volume.		

Cluster	**Solve real-world and mathematical problems involving area, surface area, and volume.**
Objectives	**Students will**
M.6.G.1	find the area of right triangles, other triangles, special quadrilaterals and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
M.6.G.2	find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\,w\,h$ and $V = B\,h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
M.6.G.3	draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
M.6.G.4	represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Grade 6	**Mathematics**			
Standard	**Statistics and Probability**			
Performance Descriptors M.PD.6.SP				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Sixth grade students at the distinguished level in mathematics: construct a viable argument as to the best average (mean, median or mode) of a set of data and justify reasoning; construct a viable argument that describes an overall pattern of the data distribution taking into account the context from which the data arose.	Sixth grade students at the above mastery level in mathematics: design and write a statistical question and justify it; critique and decide whether the observations of others make sense and ask questions to clarify.	Sixth grade students at the mastery level in mathematics: recognize that statistical questions include variability in answers; create graphical representations of data and reason abstractly and quantitatively about statistical distributions.	Sixth grade students at the partial mastery level in mathematics: calculate measures of central tendency; describe gaps, clusters and spread.	Sixth grade students at the novice level in mathematics: calculate mean of a set of data; create a line plot.

Cluster	**Develop understanding of statistical variability.**
Objectives	**Students will**
M.6.SP.1	recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question*

	because one anticipates variability in students' ages.
M.6.SP.2	understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
M.6.SP.3	recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
Cluster	**Summarize and describe distributions.**
Objectives	Students will
M.6.SP.4	display numerical data in plots on a number line, including dot plots, histograms and box plots
M.6.SP.5	summarize numerical data sets in relation to their context, such as by: a. reporting the number of observations. b. describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Mathematics - Grade 7

In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions and working with two- and three-dimensional shapes to solve problems involving area, surface area and volume; and (4) drawing inferences about populations based on samples.

1. Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

2. Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction and multiplication and division. By applying these properties and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

3. Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

4. Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 7	**Mathematics**			
Standard	**Ratio and Proportional Relationships**			
Performance Descriptors M.PD.7.RP				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Seventh grade students at the distinguished level in mathematics: make assumptions in order to simplify a complicated situation and justify those assumptions using proportional reasoning.	Seventh grade students at the above mastery level in mathematics: employ unit rates and ratios to explain proportionality in real-world situations.	Seventh grade students at the mastery level in mathematics: recognize, explain and apply proportionality to solve multi-step ratio and percent problems.	Seventh grade students at the partial mastery level in mathematics: apply unit rates to solve proportions in word problems.	Seventh grade students at the novice level in mathematics: recognize rates as ratios.

Cluster	**Analyze proportional relationships and use them to solve real-world and mathematical problems.**
Objectives	Students will
M.7.RP.1	compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.*
M.7.RP.2	recognize and represent proportional relationships between quantities. a. decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams and verbal descriptions of proportional relationships. c. represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.* d. explain what a point(x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0,0) and $(1, r)$ where r is the unit rate.
M.7.RP.3	use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Grade 7	**Mathematics**			
Standard	**The Number System**			
Performance Descriptors M.PD.7.NS				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Seventh grade students at the distinguished level in mathematics: create and present	Seventh grade students at the above mastery level in mathematics: explain and justify the	Seventh grade students at the mastery level in mathematics: apply properties of	Seventh grade students at the partial mastery level in mathematics: apply the properties of	Seventh grade students at the novice level in mathematics: recognize rational numbers

scenarios involving real-world situations to model understanding of the properties of operations and rational numbers.	selection of strategies used to solve problems involving properties of operations and rational numbers.	operations to complete computations and solve real-world problems involving rational numbers.	operations in computations involving integers.	and their additive inverses.

Cluster	**Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.**
Objectives	Students will
M.7.NS.1	apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. b. understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in real-world contexts. d. apply properties of operations as strategies to add and subtract rational numbers.
M.7.NS.2	apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real world contexts. c. apply properties of operations as strategies to multiply and divide rational numbers. d. convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
M.7.NS.3	solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.)

Grade 7	**Mathematics**			
Standard	**Expressions and Equations**			
Performance Descriptors M.PD.7.EE				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Seventh grade students at the distinguished level in mathematics:	Seventh grade students at the above mastery level in mathematics:	Seventh grade students at the mastery level in mathematics:	Seventh grade students at the partial mastery level in mathematics:	Seventh grade students at the novice level in mathematics:

create and present scenarios modeled by multiple equivalent expressions; recognize problems in real-world situations that can be modeled and solved through the application of equations and inequalities.	communicate how various properties of operations justify procedures used to simplify expressions; assess reasonableness of and justify solutions to problems involving rational numbers.	use properties of operations to make sense of and modify linear expressions in the context of a problem; generate equations and inequalities using variables to find and display solutions to multi-step problems involving rational numbers.	generate equivalent expressions through the application of properties of operations; use equations and inequalities to solve real-life problems.	simplify given expressions; solve a given equation or inequality.

Cluster	**Use properties of operations to generate equivalent expressions.**
Objectives	Students will
M.7.EE.1	apply properties of operations as strategies to add, subtract, factor and expand linear expressions with rational coefficients.
M.7.EE.2	understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."*
Cluster	**Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**
Objectives	Students will
M.7.EE.3	solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*
M.7.EE.4	use variables to represent quantities in a real-world or mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Grade 7	**Mathematics**			
Standard	Geometry			
Performance Descriptors M.PD.7.G				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Seventh grade students at	Seventh grade students at	Seventh grade students at	Seventh grade students at	Seventh grade students at

the distinguished level in mathematics: create scale models of three-dimensional geometric figures found in nature and present in the context of a real world problem; make connections between surface area and volume in real-life situations.	the above mastery level in mathematics: describe the process and justify the outcome of a geometric construction; create models to validate formulas.	the mastery level in mathematics: draw and construct geometric figures, describe geometric shapes created by transecting three-dimensional figures and use scale drawings to solve problems; use area, volumetric, and geometric formulas and geometric relationships to solve multi-step real-world problems.	the partial mastery level in mathematics: recognize quantitative characteristics of two- and three-dimensional geometric figures; given geometric formulas solve mathematical problems.	the novice level in mathematics: identify two- and three-dimensional geometric figures; use appropriate terminology to identify various geometric attributes.

Cluster	**Draw, construct and describe geometrical figures and describe the relationships between them.**
Objectives	**Students will**
M.7.G.1	solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
M.7.G.2	draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
M.7.G.3	describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
Cluster	**Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**
Objectives	**Students will**
M.7.G.4	know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
M.7.G.5	use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
M.7.G.6	solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Grade 7	**Mathematics**			
Standard	**Statistics and Probability**			
Performance Descriptors M.PD.7.SP				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Seventh grade students at	Seventh grade students at	Seventh grade students at	Seventh grade students at	Seventh grade students at

the distinguished level in mathematics:	the above mastery level in mathematics:	the mastery level in mathematics:	the partial mastery level in mathematics:	the novice level in mathematics:
identify a problem, develop an hypothesis and generate a procedure to obtain and analyze data to form and present a conclusion;	justify why conclusions based on sampling accurately describe populations;	obtain samples from a population, draw inferences and explore validity of conclusions based on the sampling;	make inferences about a given population based on sampling;	collect samples of data about a given population;
identify a problem, develop an hypothesis and generate a procedure to obtain and analyze data to form and present a conclusion between two populations;	justify the validity of comparisons of two populations;	using samples from two populations with similar variabilities, draw comparative inferences;	make connections between the data sets of two given populations;	recognize when data represents two given populations;
make and justify sound decisions based on probability.	formulate accurate explanations of experimental and theoretical probability and model results with appropriate displays.	find probability through experimentation, develop a model to find theoretical probability and determine probability for compound events.	recognize the numerical relationship among chance, odds and probability.	relate the likelihood of an event to a numerical probability.

Cluster	**Use random sampling to draw inferences about a population.**
Objectives	**Students will**
M.7.SP.1	understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
M.7.SP.2	use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*
Cluster	**Draw informal comparative inferences about two populations.**
Objectives	**Students will**
M.7.SP.3	informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*
M.7.SP.4	use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

Cluster	**Investigate chance processes and develop, use, and evaluate probability models.**
Objectives	**Students will**
M.7.SP.5	understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely and a probability near 1 indicates a likely event.
M.7.SP.6	approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*
M.7.SP.7	develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. *b.* develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*
M.7.SP.8	find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. c. design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Mathematics - Grade 8

In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity and congruence and understanding and applying the Pythagorean Theorem.

1. Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel or are the same line. Students use linear equations, systems of linear equations, linear functions and their understanding of slope of a line to analyze situations and solve problems.

2. Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

3. Students use ideas about distance and angles, how they behave under translations, rotations, reflections and dilations and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders and spheres.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 8	**Mathematics**
Standard	**The Number System**

Performance Descriptors M.PD.8.NS

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Eighth grade students at the distinguished level in mathematics: create scenarios of real-world situations that model the use of irrational numbers.	Eighth grade students at the above mastery level in mathematics: justify procedures used to determine the approximations of irrational numbers.	Eighth grade students at the mastery level in mathematics: make, use and compare approximations of irrational numbers and locate on a number line.	Eighth grade students at the partial mastery level in mathematics: differentiate between rational and irrational numbers and procedurally convert decimal expansions of rational numbers into fractions.	Eighth grade students at the novice level in mathematics: understand irrational numbers exist.

Cluster	**Know that there are numbers that are not rational, and approximate them by rational numbers**
Objectives	Students will
M.8.NS.1	know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually and convert a decimal expansion which repeats eventually into a rational number.
M.8.NS.2	use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Grade	**Mathematics**
Standard	**Expressions and Equations**

Performance Descriptors M.PD.8.EE

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Eighth grade students at the distinguished level in mathematics: within the problem situation, express numerical answers with a degree of precision appropriate for the problem context and justify reasonableness;	Eighth grade students at the above mastery level in mathematics: identify important quantities and measurements in practical situations and analyze those relationships to comfortably solve problem situations and justify conclusions;	Eighth grade students at the mastery level in mathematics generate equivalent numerical expressions from expressions involving integer exponents, use radicals to express square and cube roots and use scientific notation to express very large or small numbers and perform operations with	Eighth grade students at the partial mastery level in mathematics: convert between standard notation and scientific notation, simplify expressions involving integer exponents or square and cube roots	Eighth grade students at the novice level in mathematics: express a number in scientific notation and write expressions using exponents and radicals;

		numbers expressed in scientific notation;		
explain the meaning of and make conjectures based on linear equations, slopes, and graphs in order to make predictions;	communicate carefully formulated explanations of the proportional nature of the slope of a line;	make sense of proportional relationships and their representation in the equation and graph of a linear equation; discern a pattern between the equation y = mx + b and the graph of a line;	use a table to graph a line and determine slope;	use two points and a right triangle to find the slope of a line;
make conjectures, collect data and investigate real-world situations leading to simultaneous linear equations; interpret the results and justify the conclusions.	communicate precisely the importance of the solutions to simultaneous linear equations.	analyze linear equations and pairs of simultaneous linear equations to solve mathematical problems and interpret the results in context.	solve linear equations in two variable and find the intersection point of two lines on a graph.	solve linear equations with the variable on one side.

Cluster	**Work with radicals and integer exponents.**
Objectives	Students will
M.8.EE.1	know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
M.8.EE.2	use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
M.8.EE.3	use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9, *and determine that the world population is more than 20 times larger.*
M.8.EE.4	perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.
Cluster	**Understand the connections between proportional relationships, lines, and linear equations.**
Objectives	Students will
M.8.EE.5	graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
M.8.EE.6	use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.
Cluster	**Analyze and solve linear equations and pairs of simultaneous linear equations.**
Objectives	Students will

M.8.EE.7	solve linear equations in one variable. a. give examples of linear equations in one variable with one solution, infinitely many solutions or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
M.8.EE.8	analyze and solve pairs of simultaneous linear equations. a. understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. solve systems of two linear equations in two variables algebraically and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* c. solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Grade	**Mathematics**			
Standard	**Functions**			
Performance Descriptors M.PD.8.F				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Eighth grade students at the distinguished level in mathematics:	Eighth grade students at the above mastery level in mathematics:	Eighth grade students at the mastery level in mathematics:	Eighth grade students at the partial mastery level in mathematics:	Eighth grade students at the novice level in mathematics:
make conjectures about the form and meaning of functions in real-world situations and can comfortably predict and explain when a functional relationship will be linear or non-linear;	justify the use of a certain representation of a function and fluently transform it into an alternate representation;	fluently interpret multiple representations of functions to make sense of their properties in problem situations, discern the structure and patterns of linear and non-linear functions;	given a functional relationship, can transform it from equation to table to a graph form;	recognize when a graph is a function;
make conjectures, collect data and investigate real-world situations leading to linear functions; identify the important quantities and interpret their meaning to make predictions and justify the conclusions.	communicate precisely the meaning of rate and initial value in y= mx + b in real world situations and comfortably describe the trend of a function.	construct and model the relationships between quantities in linear functions, with emphasis on the rate and initial value, and communicate the qualitative relationship between the variables in	use a table to graph a line and determine rate and initial value.	determine whether a function is increasing or decreasing.

		functions.		

Cluster	**Define, evaluate, and compare functions.**
Objectives	**Students will**
M.8.F.1	understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (function notation not required in grade 8)
M.8.F.2	compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
M.8.F.3	interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1),(2,4) and (3,9), which are not on a straight line.*
Cluster	**Use functions to model relationships between quantities**
Objectives	**Students will**
M.8.F.4	construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
M.8.F.5	describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally

Grade	**Mathematics**
Standard	**Geometry**

Performance Descriptors M.PD.8.G

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Eighth grade students at the distinguished level in mathematics:	Eighth grade students at the above mastery level in mathematics:	Eighth grade students at the mastery level in mathematics:	Eighth grade students at the partial mastery level in mathematics:	Eighth grade students at the novice level in mathematics:
know and flexibly use the different properties of transformations to prove congruence or similarity of two-dimensional figures and of the angle relationships created when parallel lines are cut by a transversal;	discuss and informally explain how transformations can be used to determine the angle sum and exterior angles of triangles;	use the properties of transformations to understand and connect to congruence and similarity of two-dimensional figures, and to angles created by parallel lines cut by a transversal and triangles;	dilate a two-dimensional figure on the coordinate plane;	rotate, reflect, and translate two-dimensional figures;
make conjectures and plan a pathway to a solution to solve real-world right	comfortably discuss the connection of proportion, slope and right triangles in	make sense of and communicate the relationship between the	determine the length of any side of a right triangle given the length of the remaining	determine the length of the hypotenuse of a right triangle given the length of

triangle problems where the existence of triangles is not obvious; design a three-dimensional solid constructed from two or more cylinders, cones, and/or spheres with a given volume and comfortably discuss how the object meets the criteria.	the use of the Pythagorean Theorem on the coordinate plane; make a conjecture about the volume of a complex solid (consisting of cones, cylinders, and/or spheres) and plan a pathway to find the solution.	legs of a right triangle and its hypotenuse, and use the Pythagorean Theorem in real-world, coordinate plane, and mathematical problems; understand and apply the volume formulas to solve real-world and mathematical problems involving cones, cylinders and spheres.	two sides; use a given formula to find volume of a cone, cylinder, or sphere.	the two legs; identify the base shape and height of a cone or cylinder and the radius and diameter of a sphere.

Cluster	**Understand congruence and similarity using physical models, transparencies, or geometry software.**
Objectives	Students will
M.8.G.1	verify experimentally the properties of rotations, reflections and translations: a. lines are taken to lines, and line segments to line segments of the same length. b. angles are taken to angles of the same measure. c. parallel lines are taken to parallel lines.
M.8.G.2	understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
M.8.G.3	describe the effect of dilations, translations, rotations and reflections on two-dimensional figures using coordinates.
M.8.G.4	understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations and dilations; given two similar two dimensional figures, describe a sequence that exhibits the similarity between them.
M.8.G.5	use informal arguments to establish facts about the angle sum and exterior angle of triangles about the angles created when parallel lines are cut by a transversal and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*
Cluster	**Understand and apply the Pythagorean Theorem.**
Objectives	Students will
M.8.G.6	explain a proof of the Pythagorean Theorem and its converse.
M.8.G.7	apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
M.8.G.8	apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
Cluster	**Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**
Objectives	Students will
M.8.G.9	know the formulas for the volumes of cones, cylinders and spheres and use them to solve real-world and mathematical problems.

Grade	**Mathematics**			
Standard	**Statistics and Probability**			
Performance Descriptors M.PD.8.SP				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Eighth grade students at the distinguished level in mathematics: make conjectures, collect data and investigate real-world situations leading to the creation of scatter plots or two-way tables; identify the important relationships and interpret their meaning to make predictions and justify conclusions.	Eighth grade students at the above mastery level in mathematics: produce, interpret and defend the meaning of the slope and intercept of the resultant linear equation when a scatter plot of bivariate measurement data has a linear correlation.	Eighth grade students at the mastery level in mathematics: make sense of bivariate measurement data and their relationship by constructing scatter plots; communicate the meaning of the data display and if a linear relationship exists, informally fit a line to the data; make sense of categorical data by making two-way tables and communicate the meaning of any association.	Eighth grade students at the partial mastery level in mathematics: identify the trend of a scatter plot and any outliers of the data; determine if there is an association between bivariate categorical data.	Eighth grade students at the novice level in mathematics: create a scatter plot given a set of bivariate measurement data; create a two-way table given a set of bivariate categorical data.

Cluster	**Investigate patterns of association in bivariate data.**
Objectives	**Students will**
M.8.SP.1	construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association and nonlinear association.
M.8.SP.2	know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line and informally assess the model fit by judging the closeness of the data points to the line.
M.8.SP.3	use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*
M.8.SP.4	understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

Mathematics –8th grade High School Math I: Introduction

The fundamental purpose of 8th Grade Mathematics I is to formalize and extend the mathematics that students learned through the end of seventh grade. Content in this course is grouped into six critical areas, or units. The units of study deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend. 8th Grade Mathematics 1 includes an exploration of the role of rigid motions in congruence and similarity. The Pythagorean Theorem is introduced, and students examine volume relationships of cones, cylinders and spheres. 8th Grade Mathematics 1 uses properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades. The final unit in the course ties together the algebraic and geometric ideas studied. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful and logical subject that makes use of their ability to make sense of problem situations. This course differs from Mathematics I in that it contains content from 8th grade. The additional content when compared to the high school course demands a faster pace for instruction and learning.

Critical Area 1: Work with quantities and rates, including simple linear expressions and equations forms the foundation for this unit. Students use units to represent problems algebraically and graphically, and to guide the solution of problems. Student experience with quantity provides a foundation for the study of expressions, equations and functions.

Critical Area 2: Building on earlier work with linear relationships, students learn function notation and language for describing characteristics of functions, including the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically and verbally, translate between representations and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integral exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Critical Area 3: This unit builds on earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting and translating between various forms of linear equations and inequalities and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions.

Critical Area 4: This unit builds upon prior students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Critical Area 5: In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections and rotations and have used these to develop notions about what it means for two objects to be congruent.

In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Critical Area 6: Building on their work with the Pythagorean Theorem to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 8	**High School Mathematics I**			
Standard	**Relationships Between Quantities**			
Performance Descriptors M.PD.1HS8.RBQ				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Eighth Grade Math I students at the distinguished level in mathematics:	Eighth Grade Math I students at the above mastery level in mathematics:	Eighth Grade Math I students at the mastery level in mathematics:	Eighth Grade Math I students at the partial mastery level in mathematics:	Eighth Grade Math I students at the novice level in mathematics:
justify and communicate methods and conclusions;	determine reasonableness of solution;	use unit analysis to determine procedures to solve problems; express numerical answers with a degree of precision appropriate for the problem context;	select appropriate units and scale to construct a data display; select appropriate formula to solve a problem;	create data displays when given the data and a scale and use formulas;
analyze relationships within an expression to draw conclusions;	create expressions in the context of a problem;	interpret expressions in the context of a problem;	procedurally identify terms, factors and coefficients in a problem situation;	identify terms, factors and coefficients in an expression;
justify and communicate solutions and explain relationships among equations, verbal descriptions and graphs.	interpret mathematical results in the context of a situation.	analyze the relationship between quantities, recognizing constraints, in problem situations and represent the relationships	procedurally solve a system of equations; procedurally rearrange a formula.	solve equations and inequalities in one variable; identify the solution of a problem by reading a graph.

		as equations and inequalities to solve problems.		

Cluster	**Reason quantitatively and use units to solve problems.** *(Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.)*
Objectives	**Students will**
M.1HS8.RBQ.1	use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
M.1HS8.RBQ.2	define appropriate quantities for the purpose of descriptive modeling.
M.1HS8.RBQ.3	choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
Cluster	**Interpret the structure of expressions.**
Objectives	**Students will**
M.1HS8.RBQ.4	interpret expressions that represent a quantity in terms of its context.* a. interpret parts of an expression, such as terms, factors, and coefficients. b. interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P(1+r)n as the product of P and a factor not depending on P.* *(Limit to linear expressions and to exponential expressions with integer exponents).*
Cluster	**Create equations that describe numbers or relationships.**
Objectives	**Students will**
M.1HS8.RBQ.5	create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions and simple rational and exponential functions. *(Limit to linear and exponential equations and in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.)*
M.1HS8.RBQ.6	create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. *(Limit to linear and exponential equations and in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.)*
M.1HS8.RBQ.7	represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. (Limit to linear equations and inequalities.)*
M.1HS8.RBQ.8	rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R. (Limit to formulas with a linear focus.)*

Grade 8	**High School Mathematics I**			
Standard	**Linear and Exponential Relationships**			
Performance Descriptors M.PD.1HS8.LER				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Eighth Grade Math I students at the distinguished level in mathematics:	Eighth Grade Math I students at the above mastery level in mathematics:	Eighth Grade Math I students at the mastery level in mathematics:	Eighth Grade Math I students at the partial mastery level in mathematics:	Eighth Grade Math I students at the novice level in mathematics:

use technological tools to explore and deepen the understanding of concepts related to representing and solving equations and inequalities;	use estimation and other mathematical knowledge to detect possible errors in solving equations and inequalities;	strategically use appropriate tools to generate and interpret graphical representations in order to solve equations and inequalities;	procedurally solve equations and equalities by using appropriate tools to generate graphical representations;	procedurally generate graphical representations of equations and equalities;
distinguish between relations that are and are not functions and communicate reasoning;	connect and explain the relationship between a function and its graphical representation;	demonstrate understanding of the meaning of function, including as it relates to sequences and contextually use and interpret statements written in function notation;	find the range, given a function and its domain;	find an output, given a function and an input;
use the contextual situation to make and justify predictions;	use key features of a function to describe its contextual situation;	interpret key features of a function in terms of context from any of its various representations;	identify key features of a function from any of its various representations;	identify slope and intercepts, given the graph of a linear function and identify intercepts, given the graph of an exponential function;
use the contextual situations of two functions to make and justify predictions ;	use key features of two functions to compare and contrast contextual situations;	compare key features of two functions that are displayed differently;	graph a function displayed in function notation, showing its key features;	graph a linear function given its key features;
use functions to draw conclusions and further analyze relationships;	explain thought processes used in writing a function;	analyze the relationship between two quantities to write the function that models it;	determine whether a relationship is linear or exponential;	identify a linear relationship and write the function that models it;
justify and communicate generalizations; respond to the arguments of others;	analyze situations by breaking them into cases to generalize the effect of different types of transformations;	identify the connections between patterns in transformations and in the related function notation;	identify patterns between the transformations and the related function notation;	identify the effect on the y-intercept of a vertical translation on a linear function;
find and compare the	explain thought processes	distinguish between the	identify key features needed	identify a linear and an

effectiveness of two plausible solution pathways; communicate carefully formulated explanations of the parameters of a function and their relationship to the solution.	used in writing a function in problem solving; analyze the solution in the context of the problem.	identifying features of linear and exponential functions; write a function, given any of its representations, to solve a problem; interpret the contextual parameters of a function.	to write a function from a graph or table; find contextual parameters of a linear function.	exponential function from a graph or a table; create a function to model a situation.

Cluster	**Represent and solve equations and inequalities graphically.**
Objectives	Students will
M.1HS8.LER.1	understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). *(Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.)*
M.1HS8.LER.2	explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value exponential, and logarithmic functions.* *(Focus on cases where f(x) and g(x) are linear or exponential.)*
M.1HS8.LER.3	graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality) and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
Cluster	**Understand the concept of a function and use function notation.**
Objectives	Students will
M.1HS8.LER.4	understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (function notation not required in grade 8)
M.1HS8.LER.5	compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.
M.1HS8.LER.6	interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1),(2,4) and (3,9), which are not on a straight line.
M.1HS8.LER.7	understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then $f(x)$ denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f(x)$.
M.1HS8.LER.8	use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context.
M.1HS8.LER.9	recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.
M.1HS8.LER.10	construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the

	function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
M.1HS8.LER.11	describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
Cluster	**Interpret functions that arise in applications in terms of a context.**
Objectives	Students will
M.1HS8.LER.12	for a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (Focus on linear and exponential functions.)*
M.1HS8.LER.13	relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. (Focus on linear and exponential functions.)*
M.1HS8.LER.14	calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. *(Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers. Mathematics II and III will address other function types.)*
Cluster	**Analyze functions using different representations.**
Objectives	Students will
M.1HS8.LER.15	graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. graph linear and quadratic functions and show intercepts, maxima, and minima. b. graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
M.1HS8.LER.16	compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
Cluster	**Build a function that models a relationship between two quantities.** ***(Limit to linear and exponential functions.)***
Objectives	Students will
M.1HS8.LER.17	write a function that describes a relationship between two quantities. a. determine an explicit expression, a recursive process or steps for calculation from a context. b. combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
M.1HS8.LER.18	write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. *(Connect arithmetic sequences to linear functions and geometric sequences to exponential functions.)*
Cluster	**Build new functions from existing functions.** ***(Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept. While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify***

	or distinguish between the effects of the other transformations included in this standard.)
Objectives	**Students will**
M.1HS8.LER.19	identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *k f(x)*, *f(kx)*, and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
Cluster	**Construct and compare linear, quadratic, and exponential models and solve problems.**
Objectives	**Students will**
M.1HS8.LER.20	distinguish between situations that can be modeled with linear functions and with exponential functions. a. prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals. b. recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
M.1HS8.LER.21	construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
M.1HS8.LER.22	observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. *(Limit to comparisons between exponential and linear models.)*
Cluster	**Interpret expressions for functions in terms of the situation they model.** ***(Limit exponential functions to those of the form f(x) = bx + k.)***
Objectives	**Students will**
M.1HS8.LER.23	interpret the parameters in a linear or exponential function in terms of a context.

Grade 8	**High School Mathematics I**			
Standard	**Reasoning with Equations**			
Performance Descriptors M.PD.1HS8.RWE				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Eighth Grade Math I students at the distinguished level in mathematics:	Eighth Grade Math I students at the above mastery level in mathematics:	Eighth Grade Math I students at the mastery level in mathematics:	Eighth Grade Math I students at the partial mastery level in mathematics:	Eighth Grade Math I students at the novice level in mathematics:
analyze and evaluate alternate solution methods;	communicate carefully formulated explanations of the solution and the solution pathway;	use algebraic properties to justify each step in a simple equation;	procedurally write each step to solve a simple equation;	find the solution to a simple equation;
analyze and evaluate alternate solution methods;	use algebraic properties to justify each step in solving inequalities in one variable and in solving literal	solve and interpret solutions to inequalities in one variable and solve literal equations;	procedurally write steps to solve inequalities in one variable;	find the solution of an inequality in one variable with a positive coefficient;

formulate, justify and communicate a strategy for selecting the most efficient method.	equations; compare the effectiveness of two plausible solution pathways.	solve systems of equations, justifying that the solution pathway is mathematically valid.	procedurally solve systems of equations.	demonstrate that the solution to a system satisfies both equations.

Cluster	**Understand solving equations as a process of reasoning and explain the reasoning.** ***(Students should focus on and master M1.RWE.1 for linear equations and be able to extend and apply their reasoning to other types of equations in future courses. Students will solve exponential equations with logarithms in Mathematics III.)***
Objectives	**Students will**
M.1HS8.RWE.1	explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
Cluster	**Solve equations and inequalities in one variable.** ***(Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as 5x = 125 or 2x = 1/16.)***
Objectives	**Students will**
M.1HS8.RWE.2	solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
M.1HS8.RWE.3	analyze and solve pairs of simultaneous linear equations. a. understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6. c. solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.
Cluster	**Solve systems of equations.** ***(Build on student experiences graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to M1.CAG.2, which requires students to prove the slope criteria for parallel lines.)***
Objectives	**Students will**
M.1HS8.RWE.4	prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
M.1HS8.RWE.5	solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Grade 8	**High School Mathematics I**
Standard	**Descriptive Statistics** ***(Experience with descriptive statistics began as early as Grade 6. Students were expected to display numerical data and summarize it using measures of center and variability. Students will be creating scatterplots and recognizing linear trends in data. Students use***

	regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.)

Performance Descriptors M.PD.1HS8.DS

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
8th Grade Math I students at the distinguished level in mathematics:	8th Grade Math I students at the above mastery level in mathematics:	8th Grade Math I students at the mastery level in mathematics:	8th Grade Math I students at the partial mastery level in mathematics:	8th Grade Math I students at the novice level in mathematics:
analyze the validity of statistical summaries;	justify the appropriateness of the selection of data displays and statistical measures;	create single-variable data displays and identify appropriate statistical measures to compare, summarize and interpret data;	create and compare data displays;	create data displays and find statistical measures;
analyze the validity of statistical summaries;	explain the interpretation of associations and trends;	create data displays for two variables and use them to describe associations and trends;	create data displays for two variables and use them to recognize associations and trends;	create data displays for two variables;
predict and analyze the effect of a change in the data set.	make conjectures concerning correlation and causation.	interpret linear models in the context of the data and distinguish between correlation and causation.	exhibit an informal understanding of correlation coefficient.	use technology to determine the linear model and correlation coefficient.

Cluster	**Summarize, represent, and interpret data on a single count or measurement variable.** ***(In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.)***
Objectives	**Students will**
M.1HS8.DST.1	represent data with plots on the real number line (dot plots, histograms, and box plots).
M.1HS8.DST.2	use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
M.1HS8.DST.3	interpret differences in shape, center and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
M.1HS8.DST.4	construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association and nonlinear association.
M.1HS8.DST.5	know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line and informally assess the model fit by judging the closeness of the data points to the line.

M.1HS8.DST.6	use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.
M.1HS8.DST.7	understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?
Cluster	**Summarize, represent, and interpret data on two categorical and quantitative variables.** ***(Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.)***
Objectives	**Students will**
M.1HS8.DST.8	summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal and conditional relative frequencies). Recognize possible associations and trends in the data.
M.1HS8.DST.9	represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.* b. informally assess the fit of a function by plotting and analyzing residuals. *(Focus should be on situations for which linear models are appropriate.)* c. fit a linear function for scatter plots that suggest a linear association.
Cluster	**Interpret linear models.**
Objectives	**Students will**
M.1HS8.DST.10	interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. *(Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.)*
M.1HS8.DST.11	compute (using technology) and interpret the correlation coefficient of a linear fit.
M.1HS8.DST.12	distinguish between correlation and causation. *(The important distinction between a statistical relationship and a cause-and-effect relationship arises here.)*

Grade 8	**High School Mathematics I**			
Standard	**Congruence, Proof, and Constructions** ***In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.***			
Performance Descriptors M.PD.1HS8.CPC				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Eighth Grade Math I	Eighth Grade Math I	Eighth Grade Math I	Eighth Grade Math I	Eighth Grade Math I

students at the distinguished level in mathematics:	students at the above mastery level in mathematics:	students at the mastery level in mathematics:	students at the partial mastery level in mathematics:	students at the novice level in mathematics:
create conjectures regarding transformations and strategically use appropriate tools to test them;	predict the results of a sequence of transformations and strategically use appropriate tools to test the prediction;	know precise definitions and perform and describe a sequence of transformations;	perform and describe rotations and reflections;	know informal definitions and perform and describe translations;
strategically use appropriate tools to demonstrate that SSA is not sufficient criteria for triangle congruence;	strategically use appropriate tools to create counterexamples to show that AAA does not determine triangle congruence and use appropriate tools to identify SAA or AAS as criteria for triangle congruence;	use transformations of rigid motion to develop and explain the definition of congruence;	use appropriate tools to explore the requirements of triangle congruence;	recognize that geometric transformations of rigid figures will preserve congruence; identify corresponding parts of congruent figures and state that they are congruent;
identify and distinguish between correct reasoning and flawed reasoning.	formalize and defend how the steps in a construction result in the desired figure.	make formal geometric constructions with a variety of tools.	make simple, formal geometric constructions.	perform simple constructions using paper folding, reflective devises and dynamic geometric software.

Cluster	**Experiment with transformations in the plane.** ***(Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle.)***
Objectives	**Students will**
M.1HS8.CPC.1	know precise definitions of angle, circle, perpendicular line, parallel line and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
M.1HS8.CPC.2	represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
M.1HS8.CPC.3	given a rectangle, parallelogram, trapezoid or regular polygon, describe the rotations and reflections that carry it onto itself.
M.1HS8.CPC.4	develop definitions of rotations, reflections and translations in terms of angles, circles, perpendicular lines, parallel lines and line segments.

M.1HS8.CPC.5	given a geometric figure and a rotation, reflection or translation draw the transformed figure using, e.g., graph paper, tracing paper or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
Cluster	**Understand congruence in terms of rigid motions.** ***(Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.)***
Objectives	**Students will**
M.1HS8.CPC.6	use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
M.1HS8.CPC.7	use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
M.1HS8.CPC.8	explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
Cluster	***Make geometric constructions.*** ***(Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.)***
Objectives	**Students will**
M.1HS8.CPC.9	make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
M.1HS8.CPC.10	construct an equilateral triangle, a square and a regular hexagon inscribed in a circle.

Grade 8	**High School Mathematics I**
Standard	**Connecting Algebra and Geometry through Coordinates** ***Students will learn, prove and use the Pythagorean Theorem and its converse, and connect the theorem to the distance formula; use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.***

Performance Descriptors M.PD.1HS8.CAG

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Eighth Grade Math I students at the distinguished level in mathematics:	Eighth Grade Math I students at the above mastery level in mathematics:	Eighth Grade Math I students at the mastery level in mathematics:	Eighth Grade Math I students at the partial mastery level in mathematics:	Eighth Grade Math I students at the novice level in mathematics:
make conjectures and plan a pathway to a solution to ssolve real world right	knowledgeably discuss the connections of proportion, slope and right triangles in	make sense of and communicate the relationship between the	find the length of any side of a right triangle given the length of the remaining two	find the hypotenuse of a right triangle given the length of the two legs;

triangle problems where the existence of triangles is not obvious;	the use of the Pythagorean Theorem on the coordinate plane;	legs of a right triangle and its hypotenuse, and use the Pythagorean Theorem in real-world, coordinate plane and mathematical problems;	sides;	
give carefully formulated explanations showing how the distance formula is derived from the Pythagorean Theorem.	create and explain examples in the coordinate plane that depict the connection between the distance formula and Pythagorean Theorem.	use geometric definitions and the coordinate plane to prove simple theorems and to solve related problems.	find the distance between two points on a coordinate plane by using the Pythagorean Theorem; repeat this process to find the perimeter of a polygon.	use the Pythagorean Theorem to determine the length of a segment on a coordinate plane.

Cluster	**Use coordinates to prove simple geometric theorems algebraically.** ***(Reasoning with triangles in this unit is limited to right triangles; e.g., derive the equation for a line through two points using similar right triangles.)***
Objectives	**Students will**
M.1HS8.CAG.1	use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*
M.1HS8.CAG.2	prove the slope criteria for parallel and perpendicular lines; use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). *(Relate work on parallel lines to work on M1.RWE.3 involving systems of equations having no solution or infinitely many solutions.)*
M.1HS8.CAG.3	use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. *(Provides practice with the distance formula and its connection with the Pythagorean theorem.)*
M.1HS8.CAG.4	explain a proof of the Pythagorean Theorem and its converse.
M.1HS8.CAG.5	apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
M.1HS8.CAG.6	apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

High School Mathematics I

The fundamental purpose of Mathematics I is to formalize and extend the mathematics that students learned in the middle grades. The critical areas, organized into units, deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Mathematics 1 uses properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades. The final unit in the course ties together the algebraic and geometric ideas studied. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Critical Area 1: By the end of eighth grade students have had a variety of experiences working with expressions and creating equations. In this first unit, students continue this work by using quantities to model and analyze situations, to interpret expressions, and by creating equations to describe situations.

Critical Area 2: In earlier grades, students define, evaluate and compare functions and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically and verbally, translate between representations and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Critical Area 3: By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This unit builds on these earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting and translating between various forms of linear equations and inequalities and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded on understanding quantities and on relationships between them.

Critical Area 4: This unit builds upon prior students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Critical Area 5: In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections and rotations and have used these to develop notions about what it means for two objects to be congruent.

Students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Critical Area 6: Building on their work with the Pythagorean Theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 9	**High School Mathematics I**			
Standard	**Relationships Between Quantities**			
Performance Descriptors M.PD.1HS.RBQ				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Math I students at the distinguished level in mathematics: justify methods and conclusions and communicate them to others; analyze relationships within an expression to draw conclusions; justify solutions and communicate them to others; explain relationships between equations, verbal descriptions, and graphs.	Math I students at the above mastery level in mathematics: determine reasonableness of solution; create expressions in the context of a problem; interpret mathematical results in the context of a situation.	Math I students at the mastery level in mathematics: use unit analysis to determine procedures to solve problems; express numerical answers with a degree of precision appropriate for the problem context; interpret expressions in the context of a problem; analyze the relationship between quantities, recognizing constraints, in problem situations and represent them as equations and inequalities	Math I students at the partial mastery level in mathematics: select appropriate units and scale to construct a data display; select appropriate formula to solve a problem; procedurally identify terms, factors, and coefficients in a problem situation; procedurally solve a system of equations; procedurally rearrange a formula.	Math I students at the novice level in mathematics: create data displays when given the data and a scale; use formulas; identify terms, factors, and coefficients in an expression; solve equations and inequalities in one variable; identify the solution of a problem by reading a graph.

		to solve problems.		

Cluster	**Reason quantitatively and use units to solve problems.** ***(Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.)***
Objectives	**Students will**
M.1HS.RBQ.1	use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
M.1HS.RBQ.2	define appropriate quantities for the purpose of descriptive modeling.
M.1HS.RBQ.3	choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
Cluster	**Interpret the structure of expressions.**
Objectives	**Students will**
M.1HS.RBQ.4	interpret expressions that represent a quantity in terms of its context.* a. interpret parts of an expression, such as terms, factors, and coefficients. b. interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P(1+r)n as the product of P and a factor not depending on P. (Limit to linear expressions and to exponential expressions with integer exponents).*
Cluster	**Create equations that describe numbers or relationships.**
Objectives	**Students will**
M.1HS.RBQ.5	create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. (*Limit to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.)*
M.1HS.RBQ.6	create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. (*Limit to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.)*
M.1HS.RBQ.7	represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. (Limit to linear equations and inequalities.)*
M.1HS.RBQ.8	rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R. (Limit to formulas with a linear focus.)*

Grade 9	**High School Mathematics I**			
Standard	**Linear and Exponential Relationships**			
Performance Descriptors M.PD.1HS.LER				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Math I students at the distinguished level in mathematics: use technological tools to explore and deepen the understanding of concepts	Math I students at the above mastery level in mathematics:	Math I students at the mastery level in mathematics:	Math I students at the partial mastery level in mathematics:	Math I students at the novice level in mathematics:

related to representing and solving equations and inequalities;	use estimation and other mathematical knowledge to detect possible errors in solving equations and inequalities;	strategically use appropriate tools to generate and interpret graphical representations in order to solve equations and inequalities;	procedurally solve equations and equalities by using appropriate tools to generate graphical representations;	procedurally generate graphical representations of equations and equalities;
distinguish between relations that are and are not functions and communicate reasoning;	connect and explain the relationship between a function and its graphical representation;	demonstrate understanding of the meaning of function, including as it relates to sequences; contextually use and interpret statements written in function notation;	find the range, given a function (in function notation) and its domain;	find an output, given a function and an input;
use the contextual situation to make and justify predictions;	use key features of a function to describe its contextual situation;	interpret key features of a function in terms of context from any of its various representations;	identify key features of a function from any of its various representations;	identify slope and intercepts, given the graph of a linear function; identify intercepts, given the graph of an exponential function;
use the contextual situations of two functions to make and justify predictions;	use key features of two functions to compare and contrast contextual situations;	compare key features of two functions that are displayed using different representations;	graph a function displayed in function notation, showing its key features;	graph a linear function given its key features;
use functions to draw conclusions and further analyze relationships;	explain thought processes used in writing a function;	analyze the relationship between two quantities to write the function that models it;	determine whether a relationship is linear or exponential;	identify a linear relationship and write the function that models it;
justify and communicate generalizations; respond to the arguments of others;	analyze situations by breaking them into cases to generalize the effect of different types of transformations;	identify the connections between patterns in transformations and in the related function notation;	identify patterns between the transformations and the related function notation;	identify the effect on the y-intercept of a vertical translation on a linear function;
find and compare the effectiveness of two plausible solution pathways;	explain thought processes used in writing a function in problem solving;	distinguish between the identifying features of linear and exponential functions; write a function, given any	identify key features needed to write a function from a graph or table;	identify a linear and an exponential function from a graph or a table;

communicate carefully formulated explanations of the parameters of a function and their relationship to the solution.	analyze meaning of the solution in the context of the problem.	of its representations, to solve a problem; interpret the contextual parameters of a function.	find contextual parameters of a linear function.	create a function to model a situation.

Cluster	**Represent and solve equations and inequalities graphically.**
Objectives	**Students will**
M.1HS.LER.1	understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). *(Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.)*
M.1HS.LER.2	explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential and logarithmic functions.* *(Focus on cases where f(x) and g(x) are linear or exponential.)*
M.1HS.LER.3	graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
Cluster	**Understand the concept of a function and use function notation.** ***(Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses.*** ***Draw examples from linear and exponential functions. In M.1HS.LER.6, draw connection to M.1HS.LER.5, which requires students to write arithmetic and geometric sequences. Emphasize arithmetic and geometric sequences as examples of linear and exponential functions.)***
Objectives	**Students will**
M.1HS.LER.4	understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then $f(x)$ denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation $y = f(x)$.
M.1HS.LER.5	use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context.
M.1HS.LER.6	recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.
Cluster	**Interpret functions that arise in applications in terms of a context.**
Objectives	**Students will**
M.1HS.LER.7	for a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* *(Focus on linear and exponential functions.)*

M.1HS.LER.8	relate the domain of a function to its graph and where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* (*Focus on linear and exponential functions.)*
M.1HS.LER.9	calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. (*Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers. Mathematics II and III will address other function types).*
Cluster	**Analyze functions using different representations.** ***(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as y=3n and y=100·2n.)***
Objectives	**Students will**
M.1HS.LER.10	graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. graph linear and quadratic functions and show intercepts, maxima, and minima. b. graph exponential and logarithmic functions, showing intercepts and end behavior and trigonometric functions, showing period, midline and amplitude.
M.1HS.LER.11	compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
Cluster	**Build a function that models a relationship between two quantities.** ***(Limit to linear and exponential functions.)***
Objectives	**Students will**
M.1HS.LER.12	write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
M.1HS.LER.13	write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. (*Connect arithmetic sequences to linear functions and geometric sequences to exponential functions.)*
Cluster	**Build new functions from existing functions.** ***(Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept. While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.)***
Objectives	**Students will**
M.1HS.LER.14	identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
Cluster	**Construct and compare linear, quadratic, and exponential models and solve problems.**
Objectives	**Students will**
M.1HS.LER.15	distinguish between situations that can be modeled with linear functions and with exponential functions.

	a. prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals. b. recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
M.1HS.LER.16	construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship or two input-output pairs (include reading these from a table).
M.1HS.LER.17	observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. *(Limit to comparisons between exponential and linear models.)*
Cluster	**Interpret expressions for functions in terms of the situation they model.** ***(Limit exponential functions to those of the form $f(x) = bx + k$.)***
Objectives	**Students will**
M.1HS.LER.18	interpret the parameters in a linear or exponential function in terms of a context.

Grade 9	**High School Mathematics I**			
Standard	**Reasoning with Equations**			
Performance Descriptors M.PD.1HS.RWE				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Math I students at the distinguished level in mathematics: analyze and evaluate alternate solution methods; analyze and evaluate alternate solution methods; formulate, justify, and communicate a strategy for selecting the most efficient method.	Math I students at the above mastery level in mathematics: communicate carefully formulated explanations of the solution and the solution pathway ; use algebraic properties to justify each step in solving inequalities in one variable and in solving literal equations; compare the effectiveness of two plausible solution pathways.	Math I students at the mastery level in mathematics: use algebraic properties to justify each step in a simple equation; solve and interpret solutions to inequalities in one variable; solve literal equations; solve systems of equations, justifying that the solution pathway is mathematically valid.	Math I students at the partial mastery level in mathematics: procedurally write each step to solve a simple equation; procedurally write steps to solve inequalities in one variable; procedurally solve systems of equations.	Math I students at the novice level in mathematics: find the solution to a simple equation; find the solution of an inequality in one variable with a positive coefficient; demonstrate that the solution to a system satisfies both equations.
Cluster	**Understand solving equations as a process of reasoning and explain the reasoning.** ***(Students should focus on and master M1.RWE.1 for linear equations and be able to extend and apply their reasoning to other types of equations in future courses. Students will solve exponential equations with logarithms in Mathematics III.)***			

Objectives	Students will
M.1HS.RWE.1	explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
Cluster	**Solve equations and inequalities in one variable.** ***(Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as 5x = 125 or 2x = 1/16.)***
Objectives	**Students will**
M.1HS.RWE.2	solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
Cluster	**Solve systems of equations.** ***(Build on student experiences graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to M1.CAG.2, which requires students to prove the slope criteria for parallel lines.)***
Objectives	**Students will**
M.1HS.RWE.3	prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
M.1HS.RWE.4	solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Grade 9	**High School Mathematics I**			
Standard	**Descriptive Statistics**			
Performance Descriptors M.PD.1HS.DS				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Math I students at the distinguished level in mathematics: analyze the validity of statistical summaries; analyze the validity of statistical summaries;	Math I students at the above mastery level in mathematics: justify the appropriateness of the selection of data displays and statistical measures; explain the interpretation of associations and trends;	Math I students at the mastery level in mathematics: create single-variable data displays and identify appropriate statistical measures to compare, summarize, and interpret data; create data displays for two variables and use them to describe associations and trends;	Math I students at the partial mastery level in mathematics: create and compare data displays; create data displays for two variables and use them to recognize associations and trends;	Math I students at the novice level in mathematics: create data displays and find statistical measures; create data displays for two variables;

predict and analyze the effect of a change in the data set.	make conjectures concerning correlation and causation.	interpret linear models in the context of the data; distinguish between correlation and causation.	exhibit an informal understanding of correlation coefficient.	use technology to determine the linear model and correlation coefficient.

Cluster	**Summarize, represent, and interpret data on a single count or measurement variable.** ***(In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.)***
Objectives	**Students will**
M.1HS.DST.1	represent data with plots on the real number line (dot plots, histograms, and box plots).
M.1HS.DST.2	use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
M.1HS.DST.3	interpret differences in shape, center and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
Cluster	**Summarize, represent, and interpret data on two categorical and quantitative variables.** ***(Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.)***
Objectives	**Students will**
M.1HS.DST.4	summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
M.1HS.DST.5	represent data on two quantitative variables on a scatter plot and describe how the variables are related. a. fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.* b. informally assess the fit of a function by plotting and analyzing residuals. *(Focus should be on situations for which linear models are appropriate.)* c. fit a linear function for scatter plots that suggest a linear association.
Cluster	**Interpret linear models.**
Objectives	**Students will**
M.1HS.DST.6	interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. *(Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.)*
M.1HS.DST.7	compute (using technology) and interpret the correlation coefficient of a linear fit.
M.1HS.DST.8	distinguish between correlation and causation. *(The important distinction between a statistical relationship and a cause-and-effect relationship arises here.)*

Grade 9	**High School Mathematics I**			
Standard	**Congruence, Proof, and Constructions**			
Performance Descriptors M.PD.1HS.CPC				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Math I students at the	Math I students at the	Math I students at the	Math I students at the	Math I students at the

distinguished level in mathematics:	above mastery level in mathematics:	mastery level in mathematics:	partial mastery level in mathematics:	novice level in mathematics:
create conjectures regarding transformations and strategically use appropriate tools to test them;	predict the results of a sequence of transformations; strategically use appropriate tools to test the prediction;	know precise definitions; perform and describe a sequence of transformations;	perform and describe rotations and reflections;	know informal definitions; perform and describe translations;
strategically use appropriate tools to demonstrate that SSA is not sufficient criteria for triangle congruence;	strategically use appropriate tools to create counterexamples to show that AAA does not determine triangle congruence; use appropriate tools to identify SAA or AAS as criteria for triangle congruence;	use transformations of rigid motion to develop and explain the definition of congruence;	use appropriate tools to explore the requirements of triangle congruence;	recognize that geometric transformations of rigid figures will preserve congruence; identify corresponding parts of congruent figures and state that they are congruent;
identify and distinguish between correct reasoning and flawed reasoning.	formalize and defend how the steps in a construction result in the desired figure.	make formal geometric constructions with a variety of tools.	make simple formal geometric constructions.	perform simple constructions using paper folding, reflective devises, and dynamic geometric software.

Cluster	**Experiment with transformations in the plane.**
Objectives	**Students will**
M.1HS.CPC.1	know precise definitions of angle, circle, perpendicular line, parallel line and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
M.1HS.CPC.2	represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
M.1HS.CPC.3	given a rectangle, parallelogram, trapezoid or regular polygon, describe the rotations and reflections that carry it onto itself.
M.1HS.CPC.4	develop definitions of rotations, reflections and translations in terms of angles, circles, perpendicular lines, parallel lines and line segments.
M.1HS.CPC.5	given a geometric figure and a rotation, reflection or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
Cluster	**Understand congruence in terms of rigid motions.** *(Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.)*

Objectives	**Students will**
M.1HS.CPC.6	use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
M.1HS.CPC.7	use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
M.1HS.CPC.8	explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
Cluster	**Make geometric constructions.** ***(Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.)***
Objectives	**Students will**
M.1HS.CPC.9	make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
M.1HS.CPC.10	construct an equilateral triangle, a square and a regular hexagon inscribed in a circle.

Grade 9	**High School Mathematics I**
Standard	**Connecting Algebra and Geometry through Coordinates**

Performance Descriptors M.PD.1HS.CAG

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Math I students at the distinguished level in mathematics: give carefully formulated explanations showing how the distance formula is derived from the Pythagorean Theorem.	Math I students at the above mastery level in mathematics: create and explain examples in the coordinate plane that depict the connection between the distance formula and Pythagorean Theorem.	Math I students at the mastery level in mathematics: use geometric definitions and the coordinate plane to prove simple theorems and to solve related problems.	Math I students at the partial mastery level in mathematics: find the distance between two points on a coordinate plane by using the Pythagorean Theorem; repeat this process to find the perimeter of a polygon.	Math I students at the novice level in mathematics: use the Pythagorean Theorem to determine the length of a segment on a coordinate plane.

Cluster	**Use coordinates to prove simple geometric theorems algebraically.** ***(Reasoning with triangles in this unit is limited to right triangles; e.g., derive the equation for a line through two points using similar right triangles.)*** ***M.1HS.CAG.3 provides practice with the distance formula and its connection with the Pythagorean theorem.***
Objectives	**Students will**
M.1HS.CAG.1	use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*

M.1HS.CAG.2	prove the slope criteria for parallel and perpendicular lines; use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). *(Relate work on parallel lines to work on M.1HS.RWE.3 involving systems of equations having no solution or infinitely many solutions.)*
M.1HS.CAG.3	use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. *(Provides practice with the distance formula and its connection with the Pythagorean theorem.)*

High School Mathematics II: Introduction

The focus of Mathematics II is on quadratic expressions, equations, and functions; comparing their characteristics and behavior to those of linear and exponential relationships from Mathematics I as organized into six critical areas, or units. The need for extending the set of rational numbers arises and real and complex numbers are introduced so that all quadratic equations can be solved. The link between probability and data is explored through conditional probability and counting methods, including their use in making and evaluating decisions. The study of similarity leads to an understanding of right triangle trigonometry and connects to quadratics through Pythagorean relationships. Circles, with their quadratic algebraic representations, round out the course. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful and logical subject that makes use of their ability to make sense of problem situations.

Critical Area 1: Students extend the laws of exponents to rational exponents and explore distinctions between rational and irrational numbers by considering their decimal representations. In Unit 3, students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows $x+1 = 0$ to have a solution. Students explore relationships between number systems: whole numbers, integers, rational numbers, real numbers and complex numbers. The guiding principle is that equations with no solutions in one number system may have solutions in a larger number system.

Critical Area 2: Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. When quadratic equations do not have real solutions, students learn that that the graph of the related quadratic function does not cross the horizontal axis. They expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

Critical Area 3: Students begin this unit by focusing on the structure of expressions, rewriting expressions to clarify and reveal aspects of the relationship they represent. They create and solve equations, inequalities and systems of equations involving exponential and quadratic expressions.

Critical Area 4: Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

Critical Area 5: Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. It is in this unit that students develop facility with geometric proof. They use what they know about congruence and similarity to prove theorems involving lines, angles, triangles, and other polygons. They explore a variety of formats for writing proofs.

Critical Area 6: Students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants and tangents dealing with segment lengths and angle measures. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center, and the equation of a parabola with vertical axis when given an equation of its directrix and the coordinates of its focus. Given an equation of a circle, they draw the graph in the coordinate plane and apply techniques for solving quadratic equations to determine intersections between lines and circles or a parabola and between two circles. Students develop informal arguments justifying common formulas for circumference, area and volume of geometric objects, especially those related to circles.

A few (+) standards are included to increase coherence but are not expected to be addressed on high stakes assessments.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 9-10	**High School Mathematics II**			
Standard	**Extending the Number System**			
Performance Descriptors M.PD.2HS.ENS				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Mathematics II students at the distinguished level in mathematics:	Mathematics II students at the above mastery level in mathematics:	Mathematics II students at the mastery level in mathematics:	Mathematics II students at the partial mastery level in mathematics:	Mathematics II students at the novice level in mathematics:
create and simplify expressions involving rational exponents and/or radicals;	use a variety of tools to fluently simplify expressions involving rational exponents and/or radicals;	communicate how the properties of exponents justify procedures used to simplify radical expressions;	consider analogous problems in order to expand properties of integral exponents to include rational exponents;	convert between expressions containing rational exponents and expressions written in radical form;
justify the operational properties for sums and products of rational and irrational numbers;	understand and apply relationships between rational and irrational numbers;	given the domain determine the range for sums and products of rational and irrational numbers;	perform operations using rational and irrational numbers;	distinguish between rational and irrational numbers;
analyze and communicate the flexible use of algebraic properties in performing	know and flexibly use algebraic properties to perform arithmetic	use algebraic properties to perform addition, subtraction and	recognize the need to extend the number system to include complex	recognize standard form of complex numbers;

arithmetic operations over the set of complex numbers; perform and justify operations on higher order polynomials.	operations over the set of complex numbers; identify and use various methods to add, subtract and multiply polynomials.	multiplication over the set of complex numbers; use the integer system to analogously demonstrate that the polynomial system is closed with respect to addition, subtraction and multiplication.	numbers; multiply linear and quadratic polynomials.	 add and subtract linear and quadratic polynomials.

Cluster	**Extend the properties of exponents to rational exponents.**
Objectives	**Students will**
M.2HS.ENS.1	explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $(5^{1/3})^3 = 5^{(1/3)3}$ *to hold, so* $(5^{1/3})^3$ *must equal 5.*
M.2HS.ENS.2	rewrite expressions involving radicals and rational exponents using the properties of exponents.
Cluster	**Use properties of rational and irrational numbers.**
Objectives	**Students will**
M.2HS.ENS.3	explain why sums and products of rational numbers are rational, that the sum of a rational number and an irrational number is irrational and that the product of a nonzero rational number and an irrational number is irrational. *Connect to physical situations, e.g., finding the perimeter of a square of area 2.*
Cluster	**Perform arithmetic operations with complex numbers.**
Objectives	**Students will**
M.2HS.ENS.4	know there is a complex number *i* such that *i2* = −1, and every complex number has the form *a* + *bi* with *a* and *b* real.
M.2HS.ENS.5	use the relation *i2* = −1 and the commutative, associative and distributive properties to add, subtract and multiply complex numbers. *Limit to multiplications that involve* i^2 *as the highest power of i.*
Cluster	**Perform arithmetic operations on polynomials.**
Objectives	**Students will**
M.2HS.ENS.6	understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract and multiply polynomials. *Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x.*

Grade 9-10	**High School Mathematics II**			
Standard	**Quadratic Functions and Modeling**			
Performance Descriptors M.PD.2HS.QFM				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Mathematics II students at the distinguished level in mathematics:	Mathematics II students at the above mastery level in mathematics:	Mathematics II students at the mastery level in mathematics:	Mathematics II students at the partial mastery level in mathematics:	Mathematics II students at the novice level in mathematics:

interpret and refine mathematical results in contextual situations to develop appropriate models;	communicate which features of a graph or table representing quantitative relationships that suggest the existence of a function model;	sketch and interpret features of graphs and tables representing quantitative relationships;	determine the average rate of change of a function over an interval and determine the domain;	recognize functions given graphical or tabular representations;
utilize appropriate descriptions of functions to justify and challenge conclusions;	use the properties of functions to determine contextual solutions and reflect on whether the results make sense;	analyze various representations of functions to compare and contrast relationships between two functions;	graph functions and identify the components of the graph;	classify a function based on its graph;
collect data and construct models that can be described using translations and arithmetic combinations of function families;	determine how a quadratic or exponential model changes as components of the model change;	write a function describing the relationship between two quantities;	use a recursive process to represent a contextual relationship;	use the recursive process to extend the terms of a function;
apply the concept of a function and its inverse in contextual situations;	use appropriate tools to explore and analyze transformations of functions and inverses of functions;	determine the inverse of a function and the effect of various transformations of a function;	distinguish translations of functions based on their graphs;	given a table or set of ordered pairs representing a finite function, write the inverse of the function;
make a plausible argument to support the selection of appropriate models representing growth in contextual situations.	construct linear, quadratic and exponential models to compare, interpolate and extrapolate information from data representing increasing quantities.	construct models to demonstrate that a quantity increasing exponentially eventually exceeds any quantity increasing as a polynomial function.	recognize the graphical differences and similarities among linear, quadratic and exponential functions.	plot data representing linear, quadratic and exponential growth.

Cluster	**Interpret functions that arise in applications in terms of a context.**
Objectives	**Students will**
M.2HS.QFM.1	for a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
M.2HS.QFM.2	relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
M.2HS.QFM.3	calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval.

	Estimate the rate of change from a graph. *Focus on quadratic functions; compare with linear and exponential functions studied in Mathematics I.*
Cluster	**Analyze functions using different representations.**
Objectives	**Students will**
M.2HS.QFM.4	graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. graph linear and quadratic functions and show intercepts, maxima, and minima. b. graph square root, cube root and piecewise-defined functions, including step functions and absolute value functions. *Compare and contrast absolute value, step and piecewise defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range and usefulness when examining piecewise-defined functions.*
M.2HS.QFM.5	write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. use the process of factoring and completing the square in a quadratic function to show zeros, extreme values and symmetry of the graph and interpret these in terms of a context. b. use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay. M.2HS.QFM.5b extends the work begun in Mathematics I on exponential functions with integer exponents.*
M.2HS.QFM.6	compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. Focus on expanding the types of functions considered to include, linear, exponential and quadratic. Extend work with quadratics to include the relationship between coefficients and roots and that once roots are known, a quadratic equation can be factored.*
Cluster	**Build a function that models a relationship between two quantities.**
Objectives	**Students will**
M.2HS.QFM.7	write a function that describes a relationship between two quantities. a. determine an explicit expression, a recursive process or steps for calculation from a context. b. combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Focus on situations that exhibit a quadratic or exponential relationship.*
Cluster	**Build new functions from existing functions.**
Objectives	**Students will**
M.2HS.QFM.8	identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them. Focus on quadratic functions and consider including absolute value functions.*
M.2HS.QFM.9	find inverse functions:. a. solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x)= 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$. *Focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as $f(x) = x^2$, $x>0$.*
Cluster	**Construct and compare linear, quadratic, and exponential models and solve problems.**
Objectives	**Students will**

M.2HS.QFM.10	using graphs and tables, observe that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically; or (more generally) as a polynomial function.

Grade 9-10	**High School Mathematics II**			
Standard	**Expressions and Equations**			
Performance Descriptors M.PD.2HS.EE				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Mathematics II students at the distinguished level in mathematics:	Mathematics II students at the above mastery level in mathematics:	Mathematics II students at the mastery level in mathematics:	Mathematics II students at the partial mastery level in mathematics:	Mathematics II students at the novice level in mathematics:
make sense of quantities and explain their relationships in problem solving situations;	fluently manipulate the form of algebraic expressions;	deconstruct, identify and interpret parts of an algebraic expression in order to rewrite the expression;	interpret algebraic expressions as being comprised of either single or multiple components;	identify parts of algebraic expressions;
fluently explain how to produce equivalent forms of quadratic expressions in order to identify and make sense of expression properties;	routinely produce equivalent forms of quadratic expressions in order to identify and make sense of expression properties;	produce equivalent forms of quadratic expressions in order to identify and make sense of expression properties;	factor simple quadratic expressions with 1 as the coefficient of the quadratic term;	recognize equivalent forms of quadratic expressions;
analyze and explain the reasonableness and efficiency of various solution processes;	continually evaluate the reasonableness of answers as they solve problems;	create equations and inequalities to solve problems and to represent relationships between quantities;	manipulate formulas to isolate a particular variable;	graph one variable equations;
routinely analyze and explain the merits of various pathways used to quadratic equations or inequalities;	solve quadratic equations and inequalities in contextual situations;	plan, develop and apply a solution pathway to quadratic equations that may have complex solutions;	solve inequalities;	write complex solutions in standard form;
develop and defend conjectures as to when quadratic polynomials will have complex solutions;	explain why some quadratic polynomials have complex solutions;	demonstrate that polynomial identities extend analogously to include the complex number system;	solve quadratic equations;	recognize that the Fundamental Theorem of Algebra applies to quadratic polynomials;

use appropriate tools to analyze and compare solutions.	explain the meaning of the solution to a system of equations.	solve special systems of equations in two variables.	solve a system of linear equations algebraically.	solve a system of linear equations graphically.

Cluster	**Interpret the structure of expressions.**
Objectives	**Students will**
M.2HS.EE.1	interpret expressions that represent a quantity in terms of its context. a. interpret parts of an expression, such as terms, factors, and coefficients. b. interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P. *Focus on quadratic and exponential expressions. Exponents are extended from the integer exponents found in Mathematics I to rational exponents focusing on those that represent square or cube roots.*
M.2HS.EE.2	use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
M.2HS.EE.3	choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. factor a quadratic expression to reveal the zeros of the function it defines. b. complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* *It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.*
Cluster	**Create equations that describe numbers or relationships.**
Objectives	**Students will**
M.2HS.EE.4	create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions. Extend work on linear and exponential equations in Mathematics I to quadratic equations.*
M.2HS.EE.5	create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
M.2HS.EE.6	rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R. Extend to formulas involving squared variables.*
Cluster	**Solve equations and inequalities in one variable.**
Objectives	**Students will**
M.2HS.EE.7	solve quadratic equations in one variable. a. use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. *Extend to solving any quadratic equation with real coefficients,*

	including those with complex solutions.
Cluster	**Use complex numbers in polynomial identities and equations.**
Objectives	**Students will**
M.2HS.EE.8	solve quadratic equations with real coefficients that have complex solutions.
M.2HS.EE.9(+)	extend polynomial identities to the complex numbers. *For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.*
M.2HS.EE.10(+)	know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
Cluster	**Solve systems of equations.**
Objectives	**Students will**
M.2HS.EE.11	solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$. Include systems that lead to work with fractions. For example, finding the intersections between $x^2 + y^2 = 1$ and $y = (x+1)/2$ leads to the point (3/5, 4/5) on the unit circle, corresponding to the Pythagorean triple $3^2 + 4^2 = 5^2$.*

Grade 9-10	**High School Mathematics II**			
Standard	**Applications of Probability**			
Performance Descriptors M.PD.2HS.AOP				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Mathematics II students at the distinguished level in mathematics: create conceptual situations to gather, analyze, interpret and communicate the results for a set of data; compare and contrast the rules of probability and communicate the results to others; make plausible arguments as to which strategy should be used to make decisions involving probability.	Mathematics II students at the above mastery level in mathematics: utilize technology to compare and contrast types of events given sets of data; routinely interpret sets of data to determine which laws of probability apply and provides answers in terms of the given model ; utilize appropriate tools to explore and interpret data used to make fair decisions.	Mathematics II students at the mastery level in mathematics: analyze sets of data to draw conclusions about the probability type indicated; apply the rules of probability to compute probability for compound events; analyze situations and use knowledge of probability to make fair decisions.	Mathematics II students at the partial mastery level in mathematics: recognize the difference between dependent and independent events given sets of data; recognize the difference between permutations and combinations ; recognize strategies to be used to evaluate problems using probability rules.	Mathematics II students at the novice level in mathematics: identify subsets of sample spaces; understand the difference between a compound event and a simple event; identify situations where probability can be used to make fair decisions.
Cluster	**Understand independence and conditional probability and use them to interpret data.**			
Objectives	**Students will**			
M.2HS.AOP.1	describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes or as			

	unions, intersections or complements of other events ("or," "and," "not").
M.2HS.AOP.2	understand that two events *A* and *B* are independent if the probability of *A* and B occurring together is the product of their probabilities and use this characterization to determine if they are independent.
M.2HS.AOP.3	understand the conditional probability of *A* given *B* as *P(A* and *B)/P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.
M.2HS.AOP.4	construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. Build on work with two-way tables from Mathematics I to develop understanding of conditional probability and independence.*
M.2HS.AOP.5	recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*
Cluster	**Use the rules of probability to compute probabilities of compound events in a uniform probability model.**
Objectives	**Students will**
M.2HS.AOP.6	find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A* and interpret the answer in terms of the model.
M.2HS.AOP.7	apply the Addition Rule, *P(A or B) = P(A) + P(B) – P(A* and *B)*, and interpret the answer in terms of the model.
M.2HS.AOP.8 (+)	apply the general Multiplication Rule in a uniform probability model, *P(A* and *B) = P(A)P(B|A) = P(B)P(A|B)*, and interpret the answer in terms of the model.
M.2HS.AOP.9 (+)	use permutations and combinations to compute probabilities of compound events and solve problems.
Cluster	**Use probability to evaluate outcomes of decisions.**
Objectives	**Students will**
M.2HS.AOP.10 (+)	use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
M.2HS.AOP.11 (+)	analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). *This unit sets the stage for work in Mathematics III, where the ideas of statistical inference are introduced. Evaluating the risks associated with conclusions drawn from sample data (i.e. incomplete information) requires an understanding of probability concepts.*

Grade 9-10	**High School Mathematics II**			
Standard	**Similarity, Right Triangle Trigonometry, and Proof**			
Performance Descriptors M.PD.2HS.STP				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Mathematics II students at the distinguished level in mathematics:	Mathematics II students at the above mastery level in mathematics:	Mathematics II students at the mastery level in mathematics:	Mathematics II students at the partial mastery level in mathematics:	Mathematics II students at the novice level in mathematics:

routinely develop, analyze, compare, choose and defend among multiple solutions the most elegant and efficient solution to contextual problems;	routinely apply properties of dilations and transformations in contextual settings to efficiently solve problems;	verify properties of dilations and use transformations to conjecture, develop and explain properties of similar triangles ;	verify properties of dilations and use transformations to explain properties of similar triangles;	use properties of dilations to determine similar triangles;
routinely develop, challenge, explain and prove or disprove student conjectures;	fluently analyze and prove geometric theorems;	understand and use stated assumptions, definitions, and previously established results in proving geometric theorems;	use stated assumptions, definitions, and previously established results to informally justify geometric theorems;	logically order a given sequence of statements justifying geometric theorems ;
routinely develop, challenge, explain and prove or disprove student conjectures involving similarity relationships in geometric figures;	fluently analyze and prove geometric theorems involving similarity relationships in geometric figures;	understand and use stated assumptions, definitions, and previously established results in proving geometric theorems involving similarity in proving relationships in geometric figures;	informally justify geometric theorems involving similarity and relationships in geometric figures;	logically order a given sequence of statements to justify geometric theorems involving similarity;
create and explain a general process using similar triangles to analytically determine the point that will partition a segment of length d into segments whose lengths are in the ratio of a/b;	explain a how to determine the point on a segment that will partition the segment in a given ratio;	determine the point on segment that partitions the segment in a given ratio;	given a point on a segment determine the ratio of the partitions;	determine the midpoint of a segment;
develop contextual problems and explain the integral part right triangles play in the solutions;	apply right triangle solutions in contextual settings;	solve for missing parts of right triangles;	solve for missing sides in right triangles;	solve for the hypotenuse in right triangles;
develop contextual problems and explain how the Pythagorean Identity would contribute to their	analyze and explain why the Pythagorean Identity may or may not be useful in various contextual problems.	prove the Pythagorean Identity and use it to determine values of other trigonometric functions.	prove the Pythagorean Identity and use it to determine values of the sine or cosine functions.	use the Pythagorean Identity to determine values of the sine or cosine functions.

solution.				

Cluster	**Understand similarity in terms of similarity transformations**
Objectives	**Students will**
M.2HS.STP.1	verify experimentally the properties of dilations given by a center and a scale factor. a. a dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. b. the dilation of a line segment is longer or shorter in the ratio given by the scale factor.
M.2HS.STP.2	given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
M.2HS.STP.3	use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
Cluster	**Prove geometric theorems.** ***Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.***
Objectives	**Students will**
M.2HS.STP.4	prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. Implementation may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for M.2HS.C.3.*
M.2HS.STP.5	prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.*
M.2HS.STP.6	prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other and conversely, rectangles are parallelograms with congruent diagonals. Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.*
Cluster	**Prove theorems involving similarity.**
Objectives	**Students will**
M.2HS.STP.7	prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally and conversely; the Pythagorean Theorem proved using triangle similarity.*
M.2HS.STP.8	use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
Cluster	**Use coordinates to prove simple geometric theorems algebraically.**
Objectives	**Students will**
M.2HS.STP.9	find the point on a directed line segment between two given points that partitions the segment in a given ratio.
Cluster	**Define trigonometric ratios and solve problems involving right triangles.**

Objectives	Students will
M.2HS.SPT.10	understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
M.2HS.SPT.11	explain and use the relationship between the sine and cosine of complementary angles.
M.2HS.SPT.12	use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
Cluster	**Prove and apply trigonometric identities.**
Objectives	**Students will**
M.2HS.SPT.13	prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, and the quadrant of the angle. *In this course, limit θ to angles between 0 and 90 degrees. Connect with the Pythagorean theorem and the distance formula. Extension of trigonometric functions to other angles through the unit circle is included in Mathematics III.*

Grade 9-10	**High School Mathematics II**			
Standard	**Circles With and Without Coordinates**			
Performance Descriptors M.PD.2HS.C				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Mathematics II students at the distinguished level in mathematics:	Mathematics II students at the above mastery level in mathematics:	Mathematics II students at the mastery level in mathematics:	Mathematics II students at the partial mastery level in mathematics:	Mathematics II students at the novice level in mathematics:
consider other student arguments related to similarity of circles, asks useful questions to clarify or improve the arguments, and make plausible arguments as to defend whether or not a specific quadrilateral is a cyclic quadrilateral;	utilize a variety of methods to construct arguments that all circles are similar and utilize a variety of tools to visualize circle relationships;	use definitions and theorems to prove properties of circles and identify and describe relationships between and among components of circles;	make conjectures relating to the similarity of all circles and draws components of circles, circumscribed and inscribed circles;	identifies components of circles, circumscribed and inscribed circles;
build a logical progression of statements connecting the Pythagorean Theorem to the derivation of the equation of a circle;	justify the derivation of the proportionality relationship of an arc length and a radius and of inscribed and circumscribed circles of a triangle and the formula for the area of a sector;	derive the relationship of the length of the arc intercepted by an angle to the radius and the formula for the area of a sector;	use the formula for the area of a sector;	define the arc and sector of a circle and the radian measure of an angle;
construct viable arguments to prove or disprove the existence of a geometric	communicate the relationship of the Pythagorean Theorem to	derive the equations given specific components of the circle and parabola (other	describe specific components of a conic section;	identify specific components of conic sections;

figure given a set of points on a coordinate plane;	the equation of a circle and determine whether or not a given equation is the equation of a parabola;	conics are addressed in future math classes);		identify geometric figures defined by coordinates on a coordinate plane;
consider other student arguments related to geometric formulas and ask useful questions to clarify or improve the arguments;	provide justification for the algebraic proof of simple geometric theorems;	use coordinates to prove simple geometric theorems algebraically;	apply simple geometric theorems;	
contextualize when solving problems involving volume formulas.	construct viable arguments for geometric formulas and use dynamic software to draw analogies between cylinders, cones and spheres.	construct informal arguments for formulas relating to circles, cylinders, pyramids and cones and solve problems.	use formulas relating to circles, cylinders, pyramids and cones.	explain the difference between circumference, area, and volume.

Cluster	**Understand and apply theorems about circles.**
Objectives	**Students will**
M.2HS.C.1	prove that all circles are similar.
M.2HS.C.2	identify and describe relationships among inscribed angles, radii and chords. *Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
M.2HS.C.3	construct the inscribed and circumscribed circles of a triangle and prove properties of angles for a quadrilateral inscribed in a circle.
M.2HS.C.4 (+)	construct a tangent line from a point outside a given circle to the circle.
Cluster	**Find arc lengths and areas of sectors of circles.**
Objectives	**Students will**
M.2HS.C.5	derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. *Emphasize the similarity of all circles. Note that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course.*
Cluster	**Translate between the geometric description and the equation for a conic section.**
Objectives	**Students will**
M.2HS.C.6	derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
M.2HS.C.7	derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
Cluster	**Use coordinates to prove simple geometric theorems algebraically.**
Objectives	**Students will**

M.2HS.C.8	use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2). Include simple proofs involving circles.*
Cluster	**Explain volume formulas and use them to solve problems.**
Objectives	**Students will**
M.2HS.C.9	give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle and informal limit arguments. Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first.*
M.2HS.C.10	use volume formulas for cylinders, pyramids, cones and spheres to solve problems. *Volumes of solid figures scale by k^3 under a similarity transformation with scale factor k.*

High School Mathematics III (LA and STEM)

Math III LA course does not include the (+) objectives.
Math III STEM course includes objectives identified by (+) sign

It is in Mathematics III that students pull together and apply the accumulation of learning that they have from their previous courses, with content grouped into four critical areas, organized into units. They apply methods from probability and statistics to draw inferences and conclusions from data. Students expand their repertoire of functions to include polynomial, rational and radical functions. They expand their study of right triangle trigonometry to include general triangles. Finally, students bring together all of their experience with functions and geometry to create models and solve contextual problems. The Mathematical Practice Standards apply throughout each course and together with the content standards, prescribe that students experience mathematics as a coherent, useful and logical subject that makes use of their ability to make sense of problem situations.

Critical Area 1: Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

Critical Area 2: Students develop the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers and division of polynomials with long division of integers. Students identify zeros of polynomials and make connections between zeros of polynomials and solutions of polynomial equations. The standards culminates with the fundamental theorem of algebra. Rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of the standards is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

Critical Area 3: Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles. This discussion of general triangles open up the idea of trigonometry applied beyond the right triangle—that is, at least to obtuse angles. Students build on this idea to develop the notion of radian measure for angles and extend the domain of the trigonometric functions to all real numbers. They apply this knowledge to model simple periodic phenomena.

Critical Area 4: Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying functions. They identify appropriate types of functions to model a situation, adjust parameters to improve the model and compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions" is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics and geometry is applied in a modeling context.

In this course rational functions are limited to those whose numerators are of degree at most 1 and denominators of degree at most 2; radical functions are limited to square roots or cube roots of at most quadratic polynomials.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 10-11	**High School Mathematics III**			
Standard	**Inferences and Conclusions from Data**			
Performance Descriptors M.PD.3HS.IC				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Math III students at the distinguished level in mathematics:	Math III students at the above mastery level in mathematics:	Math III students at the mastery level in mathematics:	Math III students at the partial mastery level in mathematics:	Math III students at the novice level in mathematics:
communicate precisely to others why the mean and standard deviation of some data sets do not fit a normal distribution;	identify data sets whose mean and standard deviation cannot be used to fit it to a normal distribution;	use available technology to estimate appropriate outcome ranges for a given a set of normally distributed data;	interpret data using the mean and standard deviation;	calculate the mean and standard deviation for a set of normally distributed data;
build logical arguments to support your conclusions about the validity of the data-generating process;	evaluate the validity of a data-generating process;	decide if a specified model is consistent with a population based on a random sample and recognize that theoretical results may be different from empirical results;	use empirical data to create theoretical probability;	create sample data using experiments and simulations;
analyze the collection process and tables of data to justify refining the parameters;	refine the parameters to increase the validity of the prediction;	recognize that the random sample affects validity and based on its accuracy make a prediction of the outcome;	determine if the prediction of an outcome falls within the expected range;	find the expected range of values in an experiment given probability and margin of error;
communicate precisely to	recognize real world	make and analyze decisions	make conjectures using	calculate simple probability.

others how probability can be used to make decisions and analyze strategies.	situations where probability can be used to make decisions and analyze strategies.	based on probability.	probability.	

Cluster	**Summarize, represent, and interpret data on single count or measurement variable**
Objectives	**Students will**
M.3HS.IC.1	use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets and tables to estimate areas under the normal curve. *(While students may have heard of the normal distribution, it is unlikely that they will have prior experience using it to make specific estimates. Build on students' understanding of data distributions to help them see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities). Emphasize that only some data are well described by a normal distribution.)*
Cluster	**Understand and evaluate random processes underlying statistical experiments**
Objectives	**Students will**
M.3HS.IC.2	understand that statistics allows inferences to be made about population parameters based on a random sample from that population.
M.3HS.IC.3	decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? (Include comparing theoretical and empirical results to evaluate the effectiveness of a treatment.)*
Cluster	**Make inferences and justify conclusions from sample surveys, experiments, and observational studies**
Objectives	**Students will**
M.3HS.IC.4	recognize the purposes of and differences among sample surveys, experiments and observational studies; explain how randomization relates to each. *(Ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment.*
M.3HS.IC.5	use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. *(Focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness).*
M.3HS.IC.6	use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. *(Focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness).*
M.3HS.IC.7	evaluate reports based on data.
Cluster	**Use probability to evaluate outcomes of decisions**
Objectives	**Students will**
M.3HS.IC.8 (+)	use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
M.3HS.IC.9 (+)	analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). *(Extend to more complex probability models. Include situations such as those involving quality control or diagnostic tests that yields both false positive and false negative results).*

Grade 10-11	**High School Mathematics III**			
Standard	**Polynomials, Radical Relationships**			
Performance Descriptors M.PD.3HS.PR				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Math III students at the distinguished level in mathematics:	Math III students at the above mastery level in mathematics:	Math III students at the mastery level in mathematics:	Math III students at the partial mastery level in mathematics:	Math III students at the novice level in mathematics:
develop an algorithm to communicate to others the general strategies;	use a variety of general methods to factor polynomials;	factor nth degree polynomials into n factors looking for both general methods and shortcuts;	factor nth degree polynomials over the set of rational numbers;	factor quadratic over the set of rational numbers;
analyze relationships between expressions;	create expressions in the context of the situation;	interpret structure in the context of the situation;	procedurally identify terms, factors and coefficients in a problem situation;	identify terms, factors and coefficients in expression;
communicate to others why some geometric series do not have a sum;	identify real life situations that can be decontextualized using the summation formula;	derive and apply the formula for the summation of a geometric series;	use the summation formula to solve problems;	use the summation formula given the essential values;
communicate to others why the set of polynomials is not a field;	identify important similarities between the set of polynomials and the set of integers;	perform operations with polynomials applying the mathematics they know from the field properties of integers;	multiply polynomials;	add and subtract polynomials;
find polynomials that do not have zeros and explain why;	communicate to others why some polynomials do not have zeros;	determine the factors of a polynomial from the zeros and vice versa; analyze this relationship to sketch the graph;	find the factors when given a graph;	find the zeros when given the graph;
explain correspondences between combinations and Pascal's triangle;	communicate patterns that occur in the coefficients and exponents in the expansion;	solve problems using the binomial expansion;	expand binomials with coefficients of 1;	create Pascal's triangle;
communicate to others why	explain correspondences	perform operations with	multiply and divide rational	add and subtract rational expressions;

the set of rational expressions is not a field;	between the set of rational expressions and the set of rational numbers;	rational expressions using the mathematics they know from the field properties of rational numbers;	expressions;	
discern patterns in radical and rational equations with extraneous roots;	communicate to others why extraneous roots arise;	solve radical and rational equations and check their answers for extraneous roots;	solve radical and rational equations with no extraneous roots;	solve radical equations with no extraneous roots;
apply the mathematics they know to generate systems of equations given the number of solutions;	identify the possible number of solutions by looking at each function before it's graphed;	find the intersection(s) of two graphs and explain why the x coordinates are the common solutions;	graph two functions on the same coordinate system;	find the points of intersection given the graphs of two functions on the same coordinate system;
communicate to others why odd and even degree functions have different end behavior.	use a function to represent a given graph.	graph functions using important features and relationships of the graph.	identify end behavior.	find the x and y intercepts of function.

Cluster	**Use complex numbers in polynomial identities and equations.**
Objectives	**Students will**
M.3HS.PR.1 (+)	extend polynomial identities to the complex numbers. *For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.* *(Build on work with quadratics equations in Mathematics II. Limit to polynomials with real coefficients.)*
M.3HS.PR.2 (+)	know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
Cluster	**Interpret the structure of expressions.**
Objectives	**Students will**
M.3HS.PR.3	interpret expressions that represent a quantity in terms of its context. a. interpret parts of an expression, such as terms, factors, and coefficients. b. interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P(1+r)n as the product of P and a factor not depending on P.*
M.3HS.PR.4	use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. *(Extend to polynomial and rational expressions.)*
Cluster	**Write expressions in equivalent forms to solve problems.**
Objectives	**Students will**
M.3HS.PR.5	derive the formula for the sum of a geometric series (when the common ratio is not 1) , and use the formula to solve problems. *For example, calculate mortgage payments.* *(Consider extending to infinite geometric series in curricular implementations of this course description.)*
Cluster	**Perform arithmetic operations on polynomials.**
Objectives	**Students will**

M.3HS.PR.6	understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction and multiplication; add, subtract and multiply polynomials. *(Extend beyond the quadratic polynomials found in Mathematics II.)*
Cluster	**Understand the relationship between zeros and factors of polynomials.**
Objectives	**Students will**
M.3HS.PR.7	know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.
M.3HS.PR.8	identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.
Cluster	**Use polynomial identities to solve problems.**
Objectives	**Students will**
M.3HS.PR.9	prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples. (This cluster has many possibilities for optional enrichment, such as relating the example to the solution of the system $u^2+v^2=1$, $v = t(u+1)$, relating the Pascal triangle property of binomial coefficients to (x+y)n+1 = (x+y)(x+y)n, deriving explicit formulas for the coefficients, or proving the binomial theorem by induction.)*
M.3HS.PR.10(+)	know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle
Cluster	**Rewrite rational expressions**
Objectives	**Students will**
M.3HS.PR.11	rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. *(The limitations on rational functions apply to the rational expressions.)*
M.3HS.PR.12(+)	understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply and divide rational expressions. *(Requires the general division algorithm for polynomials).*
Cluster	**Understand solving equations as a process of reasoning and explain the reasoning.**
Objectives	**Students will**
M.3HS.PR.13	solve simple rational and radical equations in one variable and give examples showing how extraneous solutions may arise. *(Extend to simple rational and radical equations)*
Cluster	**Represent and solve equations and inequalities graphically.**
Objectives	**Students will**
M.3HS.PR.14	explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential and logarithmic functions. *(Include combinations of linear, polynomial, rational, radical, absolute value, and exponential functions.)*
Cluster	**Analyze functions using different representations.**
Objectives	**Students will**
M.3HS.PR.15	graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end

	behavior. *(Relate to the relationship between zeros of quadratic functions and their factored forms.)*

Grade 10-11	**High School Mathematics III**
Standard	**Trigonometry of General Triangles and Trigonometric Functions**

Performance Descriptors M.PD.3HS.TF

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Math III students at the distinguished level in mathematics:	Math III students at the above mastery level in mathematics:	Math III students at the mastery level in mathematics:	Math III students at the partial mastery level in mathematics:	Math III students at the novice level in mathematics:
justify and communicate to others the reason why some given measurements do not determine a unique triangle;	identify real life applications that can be represented using triangles, decontextualize (represent symbolically) the situation, and solve using trigonometry;	derive and apply the appropriate formulas to calculate accurately and efficiently the unknown measurements of any triangle;	procedurally use trigonometric formulas to solve triangles;	use the definition of sine and cosine to solve right triangles;
justify to others the special unit circle values;	derive the special unit circle values;	use the radian measures of the unit circle in practical situations to explain how to calculate the arc length and determine trigonometric values;	use the radian measures of the unit circle to calculate the arc length and determine trigonometric values;	memorize the special unit circle values;
determine an algorithm for finding accurate parameters of a trigonometric function that fits the data and justify their conclusions.	choose appropriate tools to attend to the meaning of quantities that determine accurate parameters for the trigonometric function that fits the data.	use technology to explore and choose trigonometric functions to model periodic data using accurate parameters.	write the equation of a trigonometric function given the graph.	find the amplitude, period, and midline, given the graph of a trigonometric function.

Cluster	**Apply trigonometry to general triangles.**
Objectives	**Students will**
M.3HS.TF.1 (+)	derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
M.3HS.TF.2 (+)	prove the Laws of Sines and Cosines and use them to solve problems. *(With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles.)*
M.3HS.TF.3 (+)	understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
Cluster	**Extend the domain of trigonometric functions using the unit circle.**

Objectives	**Students will**
M.3HS.TF.4	understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
M.3HS.TF.5	explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
Cluster	**Model periodic phenomena with trigonometric functions.**
Objectives	**Students will**
M.3HS.TF.6	choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Grade 10-11	**High School Mathematics III**			
Standard	**Mathematical Modeling**			
Performance Descriptors M.PD.3HS.MM				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Math III students at the distinguished level in mathematics: communicate to others the relationships among the words, graphs and equations; contextualize and decontextualize a real world situation given the key features of a function; contextualize a real world situation given different representations of a function; interpret adding a constant function to a given function in a real world situation; interpret the transformation of a function in a real world situation;	Math III students at the above mastery level in mathematics: interpret mathematical results in the context of the situation; contextualize a real world situation given a function; decontextualize a real world situation given different representations of a function; find a pattern and communicate to others how the graph changes by adding a constant function; find a pattern and communicate to others how the graph changes by using transformations;	Math III students at the mastery level in mathematics: create equations and inequalities from words and use them to solve problems; interpret the important features of a function in the context of the situation; analyze different representations of a function in the context of the situation; build functions by adding a constant function to another function and search for regularity or trends; build functions by using transformations and search for regularity or trends;	Math III students at the partial mastery level in mathematics: create equations and inequalities in one variable from words; interpret rate of change of a function; analyze rate of change in different representations of a function; build functions by adding a constant function to another function, where the constant is a positive integer; find the equation of a function given a description of the transformation;	Math III students at the novice level in mathematics: isolate one variable in a formula; interpret the key features of a linear function; analyze different representations of a linear function; identify a constant function; graph a function given a description of the transformation;

use inverse relationships to explain the graphs of exponential and logarithmic functions; classify three-dimensional objects by their cross-sections; communicate to others the reason that ratios and proportions can be used to find parts of similar figures.	use properties of logarithms to transform an expression; communicate the relationship between the area of the cross section and the volume of the object; communicate to others the relationships between the ratios of sides, areas and volumes of similar figures.	use logarithms to represent exponential models and evaluate the logarithms using technology; visualize relationships between cross-section and three-dimensional objects; use ratios and proportions of similar figures to solve real world problems.	convert from an exponential equation to a logarithmic equation; describe the three-dimensional object by its cross-section; make a scale drawing on topographic grid paper.	use technology to evaluate base 10 or base e logarithms; describe cross-section and three-dimensional objects; solve proportions.

Cluster	**Create equations that describe numbers or relationships.**
Objectives	**Students will**
M.3HS.MM.1	create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions. (Use all available types of functions to create such equations, including root functions, but constrain to simple cases.)*
M.3HS.MM.2	create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. *(While functions will often be linear, exponential or quadratic the types of problems should draw from more complex situations than those addressed in Mathematics I. For example, finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line.)*
M.3HS.MM.3	represent constraints by equations or inequalities and by systems of equations and/or inequalities and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
M.3HS.MM.4	rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R. (The example given applies to earlier instances of this standard, not to the current course.)*
Cluster	**Interpret functions that arise in applications in terms of a context.**
Objectives	**Students will**
M.3HS.MM.5	for a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (Emphasize the selection of a model function based on behavior of data and context.)*
M.3HS.MM.6	relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

M.3HS.MM.7	calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
Cluster	**Analyze functions using different representations.**
Objectives	**Students will**
M.3HS.MM.8	graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. graph square root, cube root and piecewise-defined functions, including step functions and absolute value functions. b. graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline and amplitude. *(Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.)*
M.3HS.MM.9	write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
M.3HS.MM.10	compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
Cluster	**Build a function that models a relationship between two quantities.**
Objectives	**Students will**
M.3HS.MM.11	write a function that describes a relationship between two quantities. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. (Develop models for more complex or sophisticated situations than in previous courses.)*
Cluster	**Build new functions from existing functions.**
Objectives	**Students will**
M.3HS.MM.12	identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them. (Use transformations of functions to find more optimum models as students consider increasingly more complex situations. Note the effect of multiple transformations on a single function and the common effect of each transformation across function types. Include functions defined only by graph.)*
M.3HS.MM.13	find inverse functions. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2\ x3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$. (Extend to simple rational, simple radical, and simple exponential functions.)*
Cluster	**Construct and compare linear, quadratic, and exponential models and solve problems.**
Objectives	**Students will**
M.3HS.MM.14	for exponential models, express as a logarithm the solution to $a\ b^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. *(Consider extending this unit to include the relationship between properties of logarithms and properties of exponents, such as the connection between the properties of exponents and the basic logarithm property that log xy = log x + log y.)*
Cluster	**Visualize relationships between two dimensional and three-dimensional objects.**
Objectives	**Students will**

M.3HS.MM.15	identify the shapes of two-dimensional cross-sections of three dimensional objects and identify three-dimensional objects generated by rotations of two-dimensional objects.
Cluster	**Apply geometric concepts in modeling situations.**
Objectives	**Students will**
M.3HS.MM.16	use geometric shapes, their measures and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
M.3HS.MM.17	apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
M.3HS.MM.18	apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

High School Mathematics Math IV

The fundamental purpose of Mathematics IV is to generalize and abstract learning accumulated through previous courses and to provide the final springboard to calculus. Students take an extensive look at the relationships among complex numbers, vectors, and matrices. They build on their understanding of functions, analyze rational functions using an intuitive approach to limits and synthesize functions by considering compositions and inverses. Students expand their work with trigonometric functions and their inverses and complete the study of the conic sections begun in Mathematics II. They enhance their understanding of probability by considering probability distributions. Previous experiences with series are augmented.

Building Relationships among Complex Numbers, Vectors, and Matrices
Students analyze complex numbers geometrically and draw on analogies between complex numbers and vector quantities. Students utilize vectors to model physical phenomena and solve related problems. Vectors are then generalized to matrices, with emphasis on utilizing matrices in transformations and applications. Matrices are additionally developed as a tool for solving systems of equations. The fundamental idea of this unit is the development of new arithmetic operations that have a commonality with each other and are a precursor to the algebraic thinking in linear algebra.

Analysis and Synthesis of Functions
Students have previously analyzed the graphs of polynomial functions. Students extend their experiences to describe the properties of rational functions. They also explore composition of functions to generalize the concept of inverses developed in Math II.

Trigonometric and Inverse Trigonometric Functions of Real Numbers
Students advance their thinking about trigonometric functions to a more abstract level. They blend the more concrete trigonometric ideas developed in Math II and III with the general function concepts in Unit 2.

Derivations in Analytic Geometry
Students extend their understanding of the definitions of the conic sections to include ellipses and hyperbolas and use them to model physical phenomena. Students develop informal arguments justifying the formulas for the volumes of more complex solids.

Modeling with Probability
Students interpret geometrically probability concepts developed since the middle grades. They then examine the role of expected value in decision making.

Series and Informal Limits
Students develop sigma notation and infinite geometric series building on ideas from Math I and Math III. This unit provides an opportunity for students to deepen their informal understanding of limits while developing formulas used in Calculus.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 11-12	High School Mathematics IV			
Standard	**Building Relationships among Complex Numbers, Vectors, and Matrices**			
Performance Descriptors M.PD.4HS.CVM				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Math IV students at the distinguished level in mathematics:	Math IV students at the above mastery level in mathematics:	Math IV students at the mastery level in mathematics:	Math IV students at the partial mastery level in mathematics:	Math IV students at the novice level in mathematics:
justify algebraically the procedures used to perform operations on complex numbers;	justify geometrically the procedures used to perform operations on complex numbers;	make sense of all four binary operations of complex numbers in either form;	procedurally divide complex numbers involving integers;	procedurally add, subtract, and multiply complex numbers involving integers;
justify powers and roots geometrically in terms of DeMoivre's Theorem;	model binary operations geometrically in terms of DeMoivre's Theorem;	determine appropriate forms and manipulate with accuracy and precision;	represent numbers on the complex plane in polar form;	represent numbers on the complex plane in rectangular form;
interpret nonlinear motion at constant speed as acceleration;	apply vector quantities to application problems;	represent abstract situations involving vectors symbolically;	identify and calculate the magnitude and direction of vector quantities;	procedurally calculate vector components from their coordinates;
construct an argument to justify how vector resultants model physical situations;	verify vector operations both geometrically and algebraically;	make sense of a geometric representation of vector operations;	understand that positive scalar multiplication changes the magnitude, but not the direction of a vector;	perform addition and subtraction of vectors both geometrically and algebraically;
compare the algebraic properties of complex numbers, vectors, and matrices;	justify the field properties that are satisfied by matrices and those that are not;	interpret transformations in the plane in terms of multiplication by 2 x 2 matrices; recognize the role of the determinant in finding	recognize the similarities of the zero and identity matrices to the real numbers 0 and 1;	procedurally perform addition, subtraction, multiplication, and scalar multiplication involving matrices;

decontextualize matrices as a tool in solving systems of equations.	model complex applications using matrix equations.	area; create a coherent representation of any system of linear equations as a matrix .	procedurally solve systems in two unknowns using the inverse of a matrix.;	procedurally solve systems using technology.

Cluster	**Perform arithmetic operations with complex numbers**
Objectives	**Students will**
M.4HS.CVM.1	find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. (*Instructional Note: In Math II students extended the number system to include complex numbers and performed the operations of addition, subtraction, and multiplication.)*
Cluster	**Represent complex numbers and their operations on the complex plane.**
Objectives	**Students will**
M.4HS.CVM.2	represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
M.4HS.CVM.3	represent addition, subtraction, multiplication and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\,i)^3 = 8$ *because* $(-1 + \sqrt{3}\,i)$ *has modulus* 2 *and argument* 120°.
M.4HS.CVM.4	calculate the distance between numbers in the complex plane as the modulus of the difference and the midpoint of a segment as the average of the numbers at its endpoints.
Cluster	**Represent and model with vector quantities.**
Objectives	**Students will**
M.4HS.CVM.5	recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments and use appropriate symbols for vectors and their magnitudes (e.g., v, $\lvert v\rvert$, $\lVert v\rVert$, v). *(Instructional Note: This is the student's first experience with vectors. The vectors must be represented both geometrically and in component form with emphasis on vocabulary and symbols.)*
M.4HS.CVM.6	find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
M.4HS.CVM.7	solve problems involving velocity and other quantities that can be represented by vectors.
Cluster	**Perform operations on vectors.**
Objectives	**Students will**
M.4HS.CVM.8	add and subtract vectors. a. add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order and perform vector subtraction component-wise.
M.4HS.CVM.9	multiply a vector by a scalar. a. represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar

	multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. b. compute the magnitude of a scalar multiple ***cv*** using $\|\|cv\|\| = \|c\|v$. Compute the direction of ***cv*** knowing that when $\|c\|v \neq 0$, the direction of ***cv*** is either along ***v*** (for $c > 0$) or against ***v*** (for $c < 0$).
Cluster	**Perform operations on matrices and use matrices in applications.**
Objectives	**Students will**
M.4HS.CVM.10	use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
M.4HS.CVM.11	multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
M.4HS.CVM.12	add, subtract and multiply matrices of appropriate dimensions.
M.4HS.CVM.13	understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. *(Instructional Note: This is an opportunity to view the algebraic field properties in a more generic context, particularly noting that matrix multiplication is not commutative.)*
M.4HS.CVM.14	understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
M.4HS.CVM.15	multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
M.4HS.CVM.16	work with 2 × 2 matrices as transformations of the plane and interpret the absolute value of the determinant in terms of area. *(Instructional Note: Matrix multiplication of a 2 x 2 matrix by a vector can be interpreted as transforming points or regions in the plane to different points or regions. In particular a matrix whose determinant is 1 or -1 does not change the area of a region.)*
Cluster	**Solve systems of equations**
Objectives	**Students will**
M.4HS.CVM.17	represent a system of linear equations as a single matrix equation in a vector variable.
M.4HS.CVM.18	find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). *Instructional Note: Students have earlier solved two linear equations in two variables by algebraic methods.*

Grade 11-12	**High School Mathematics IV**			
Standard	**Analysis and Synthesis of Functions**			
Performance Descriptors M.PD.4HS.ASF				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Math IV students at the distinguished level in mathematics: utilize informal limits to analyze oblique asymptotes and continuity; model applications using composition of functions;	Math IV students at the above mastery level in mathematics: utilize informal limits to analyze horizontal asymptotes; recognize the relationships of domains and ranges in	Math IV students at the mastery level in mathematics: utilize informal limits to analyze vertical asymptotes; make sense of composition relationships in problem	Math IV students at the partial mastery level in mathematics: procedurally sketch polynomial and rational functions by hand; understand composition as a function of the result of	Math IV students at the novice level in mathematics: procedurally graph polynomial and rational functions using technology; compute procedurally the composition of two

model applications using exponential and logarithmic functions.	the functions; recognize the similar behavior of all inverse functions.	situations; restrict the domain of a function to create an invertible function and make sense of the inverse relationship between exponential and logarithmic functions.	another function; procedurally show that two functions are inverses by using composition.	functions; determine inverses of a function by interchanging x and y.

Cluster	**Analyze functions using different representations.**
Objectives	**Students will**
M.4HS.ASF.1	graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. graph rational functions, identifying zeros and asymptotes when suitable factorizations are available and showing end behavior. *This is an extension of M.3HS.MM8 that develops the key features of graphs with the exception of asymptotes. (Instructional Note: Students examine vertical, horizontal and oblique asymptotes by considering limits. Students should note the case when the numerator and denominator of a rational function share a common factor.)* b. utilize an informal notion of limit to analyze asymptotes and continuity in rational functions. *(Instructional Note: Although the notion of limit is developed informally, proper notation should be followed.)*
Cluster	**Build a function that models a relationship between two quantities.**
Objectives	**Students will**
M.4HS.ASF.2	write a function that describes a relationship between two quantities, including composition of functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
Cluster	**Build new functions from existing functions**
Objectives	**Students will**
M.4HS.ASF.3	find inverse functions. *This is an extension of M.3HS.MM.13 which introduces the idea of inverse functions.* a. verify by composition that one function is the inverse of another. *b.* read values of an inverse function from a graph or a table, given that the function has an inverse. *(Instructional Note: Students must realize that inverses created through function composition produce the same graph as reflection about the line y = x.)* *c.* produce an invertible function from a non-invertible function by restricting the domain. *(Instructional Note: Systematic procedures must be developed for restricting domains of non-invertible functions so that their inverses exist.)*
M.4HS.ASF.4	understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Grade 11-12	**High School Mathematics IV**
Standard	**Trigonometric and Inverse Trigonometric Functions of Real Numbers**
Performance Descriptors M.PD.4HS.TF	

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Math IV students at the distinguished level in mathematics:	Math IV students at the above mastery level in mathematics:	Math IV students at the mastery level in mathematics:	Math IV students at the partial mastery level in mathematics:	Math IV students at the novice level in mathematics:
justify and explain to others the symmetric properties of the trigonometric functions;	develop simplifying strategies for determining the special unit circle values;	make sense of the symmetry, periodicity, and special values of trigonometric functions using the unit circle;	calculate the sine, cosine, and tangent of special angles;	convert between degree and radian measure;
justify the use of trigonometric functions in modeling periodic phenomena;	synthesize algebraic and trigonometric methods of solving equations;	solve trigonometric equations in modeling contexts;	understand the relationship between trigonometric functions and their inverses;	using technology, calculate values of inverse trigonometric functions;
communicate to others methods of proving trigonometric identities;	communicate to others the use of trigonometric identities in problem solving situations;	prove trigonometric identities and apply them problem solving situations;	understand the structure of the addition and subtraction trigonometric identities;	check the reasonableness of the addition and subtraction trigonometric identities by substitution of numerical values;
model periodic phenomena in terms of trigonometric functions involving phase shifts.	compare phase shifts in trigonometric functions to transformations of other functions.	calculate phase shift of a trigonometric function with accuracy and precision.	recognize the existence of phase shift, given the trigonometric function in algebraic form.	recognize the existence of phase shift, given the graph of a trigonometric function.

Cluster	**Extend the domain of trigonometric functions using the unit circle.**
Objectives	**Students will**
M.4HS.TF.1	use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number. *(Instructional Note: Students use the extension of the domain of the trigonometric functions developed in Math III to obtain additional special angles and more general properties of the trigonometric functions.)*
M.4HS.TF.2	use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
Cluster	**Model periodic phenomena with trigonometric functions.**
Objectives	**Students will**
M.4HS.TF.3	understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
M.4HS.TF.4	use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. *(Instructional Note: Students should draw analogies to the work with inverses in the previous unit.)*

M.4HS.TF.5	solve more general trigonometric equations. *For example* $2\sin^2 x + \sin x - 1 = 0$ *can be solved using factoring.*
Cluster	**Prove and apply trigonometric identities.**
Objectives	Students will
M.4HS.TF.6	prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
Cluster	**Apply transformations of function to trigonometric functions.**
Objectives	Students will
M.4HS.TF.7	graph trigonometric functions showing key features, including phase shift. *(Instructional Note: In Math III, students graphed trigonometric functions showing period, amplitude and vertical shifts.)*

Grade 11-12	**High School Mathematics IV**
Standard	**Derivations in Analytic Geometry**

Performance Descriptors M.PD.4HS.AG

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Math IV students at the distinguished level in mathematics:	Math IV students at the above mastery level in mathematics:	Math IV students at the mastery level in mathematics:	Math IV students at the partial mastery level in mathematics:	Math IV students at the novice level in mathematics:
model physical phenomena using ellipses and hyperbolas;	apply ellipses and hyperbolas to physical phenomena given the model;	make sense of the derivations of the equations of an ellipse and a hyperbola;	distinguish between ellipses and hyperbolas, given their equations in standard form;	recognize ellipses and hyperbolas as conic sections;
construct an argument, using Cavalieri's principle, to develop the volume formulas for various solids.	communicate to others the meaning of Cavalieri's principle.	understand the meaning of Cavalieri's principle, including non-congruent cross sections.	visualize Cavalieri's principle for congruent cross sections.	recognize that skewing a prism does not change its volume.

Cluster	**Translate between the geometric description and the equation for a conic section.**
Objectives	Students will
M.4HS.AG.1	derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. *(Instructional Note: In Math II students derived the equations of circles and parabolas. These derivations provide meaning to the otherwise arbitrary constants in the formulas.)*
Cluster	**Explain volume formulas and use them to solve problems.**
Objectives	Students will
M.4HS.AG.2	give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. *(Instructional Note: Students were introduced to Cavalieri's principle in Math II.)*

Grade 11-12	**High School Mathematics IV**
Standard	**Modeling with Probability**

Performance Descriptors M.PD.4HS.MP

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Math IV students at the distinguished level in mathematics:	Math IV students at the above mastery level in mathematics:	Math IV students at the mastery level in mathematics:	Math IV students at the partial mastery level in mathematics:	Math IV students at the novice level in mathematics:
model applications using probability distributions;	analyze the shape of a probability distribution in terms of its data;	understand probability distributions and expected value as a mean;	calculate theoretical probabilities and graph their probability distributions;	create basic sample spaces;
construct an argument to justify whether a decision-making strategy based on expected value is appropriate.	compare or contrast decision-making strategies based upon expected value.	create a decision-making strategy based upon expected values.	assign random variables to calculate an expected value.	given the random variable, calculate expected value.

Cluster	**Calculate expected values and use them to solve problems.**
Objectives	**Students will**
M.4HS.MP.1	define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. *(Instructional Note: Although students are building on their previous experience with probability in middle grades and in Math II and III, this is their first experience with expected value and probability distributions.)*
M.4HS.MP.2	calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
M.4HS.MP.3	develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*
M.4HS.MP.4	develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households? (Instructional Note: It is important that students can interpret the probability of an outcome as the area under a region of a probability distribution graph.)*
Cluster	**Use probability to evaluate outcomes of decisions.**
Objectives	**Students will**
M.4HS.MP.5	weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast food restaurant.* b. evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*

Grade 11-12	**High School Mathematics IV**
Standard	**Series and Informal Limits**

Performance Descriptors M.PD.4HS.SL				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Math IV students at the distinguished level in mathematics: construct general arguments using mathematical induction; make sense intuitively of the concept of limit of a series.	Math IV students at the above mastery level in mathematics: construct an argument using mathematical induction for summation formulas; construct models using geometric series.	Math IV students at the mastery level in mathematics: apply mathematical induction to prove summation formulas; understand that an infinite sum of positive numbers can converge.	Math IV students at the partial mastery level in mathematics: convert an expanded series to summation notation and expand a series given in summation notation; describe geometrically a converging geometric series.	Math IV students at the novice level in mathematics: manipulate summation notation; compute the sum of an infinite geometric series.

Cluster	**Use sigma notations to evaluate finite sums.**
Objectives	**Students will**
M.4HS.SL.1	develop sigma notation and use it to write series in equivalent form. For example, write $\sum_{i=1}^{n} 3i^2 + 7$ as $3\sum_{i=1}^{n} i^2 + 7\sum_{i=1}^{n} 1$.
M.4HS.SL.2	apply the method of mathematical induction to prove summation formulas. For example, verify that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$. *(Instructional Note: Some students may have encountered induction in Math III in proving the Binomial Expansion Theorem, but for many this is their first experience.)*
Cluster	**Extend geometric series to infinite geometric series.**
Objectives	**Students will**
M.4HS.SL.3	develop intuitively that the sum of an infinite series of positive numbers can converge and derive the formula for the sum of an infinite geometric series. *(Instructional Note: In Math I, students described geometric sequences with explicit formulas. Finite geometric series were developed in Math III.)*
M.4HS.SL.4	apply infinite geometric series models. *For example, find the area bounded by a Koch curve. (Instructional Note: Rely on the intuitive concept of limit developed in unit 2 to justify that a geometric series converges if and only if the ratio is between -1 and 1.)*

High School Mathematics STEM READINESS

This course is designed for students who have completed the Math III (LA) course and subsequently decided they are interested in pursuing a STEM career. It includes standards that would have been covered in Math III (STEM) but not in Math III (LA) (i.e. standards in the CCSS document that are marked with a "+"), selected topics from the suggested CCSS Math IV course, and topics drawing from standards covered in Math I and Math II as needed for coherence.

Arithmetic and algebra of complex numbers
This unit reviews the arithmetic of complex numbers with the goal of extending algebraic ideas to the complex number system, for example polynomial identities, the quadratic formula and the Fundamental Theorem of Algebra. Students identify zeros of polynomials, including complex zeros of quadratic polynomials and make connections between complex zeros of polynomials and solutions of polynomial equations. A central theme of this unit is that the arithmetic and algebra of expressions involving rational numbers is governed by the same rules as the arithmetic and algebra of real numbers.

Polynomials, rational, and radical relationships
Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers, which is governed by the same rules as the arithmetic of complex numbers.

Probability for decisions
Students see the role that randomness and careful design play in the conclusions that can be drawn. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment.

Trigonometry of general triangles
Students develop the Laws of Sines and Cosines in order to find missing measures of general (acute, right, or obtuse) triangles. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles. Area is another measure that can be included. This discussion of general triangles opens up the idea of trigonometry applied beyond the right triangle—that is, at least to obtuse angles. Students build on this idea to develop the notion of radian measure for angles and extend the domain of the trigonometric functions to all real numbers. They apply this knowledge to model simple periodic phenomena.

Functions and modeling
Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions and trigonometric functions to include solving exponential equations with logarithms and giving general solutions incorporating periodicity. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations of a graph always have the same effect regardless of the type of the underlying functions. They identify appropriate

types of functions to model a situation, adjust parameters to improve the model and compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 11-12	**High School Mathematics STEM Readiness**			
Standard	**Arithmetic and algebra of complex numbers**			
Performance Descriptors M.PD.SRM.CN				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
STEM Readiness students at the distinguished level in mathematics: justify algebraically using the field properties the procedures used to perform operations on complex numbers; develop and defend conjectures as to when quadratic polynomials will have complex solutions.	STEM Readiness students at the above mastery level in mathematics: justify geometrically the procedures used to perform operations on complex numbers; explain why some quadratic polynomials have complex solutions.	STEM Readiness students at the mastery level in mathematics: manipulate expressions involving complex numbers using modulus and all four binary operations and make sense of how the conjugate is used to perform division; demonstrate that polynomial identities extend analogously to include the complex number system.	STEM Readiness students at the partial mastery level in mathematics: procedurally evaluate expressions involving complex numbers of the form $a+bi$ with a and b integers; solve quadratic equations.	STEM Readiness students at the novice level in mathematics: procedurally evaluate expressions involving complex numbers of the form $a+bi$ with a and b integers involving addition, subtraction, and multiplication; recognize that the Fundamental Theorem of Algebra applies to quadratic polynomials.

Cluster	**Perform arithmetic operations with complex numbers**
Objectives	**Students will**
M.SRM.CN.1	know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
M.SRM.CN.2	use the relation $i^2 = -1$ and the commutative, associative and distributive properties to add, subtract and multiply complex numbers.
M.SRM.CN.3	find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
Cluster	**Represent complex numbers and their operations on the complex plane**
Objectives	**Students will**
M.SRM.CN.4	represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers) and explain why the rectangular and polar forms of a given complex number represent the same number.

M.SRM.CN.5	represent addition, subtraction, multiplication and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\,i)^3 = 8$ *because* $(-1 + \sqrt{3}\,i)$ *has modulus 2 and argument 120°.*
M.SRM.CN.6	calculate the distance between numbers in the complex plane as the modulus of the difference and the midpoint of a segment as the average of the numbers at its endpoints.
Cluster	**Use complex numbers in polynomial identities and equations**
Objectives	**Students will**
M.SRM.CN.7	solve quadratic equations with real coefficients that have complex solutions.
M.SRM.CN.8	extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.
M.SRM.CN.9	know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Grade 11-12	**High School Mathematics STEM Readiness**
Standard	**Polynomials, rational, and radical relationships**

Performance Descriptors M.PD.SRM.PRR

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
STEM Readiness students at the distinguished level in mathematics: explain correspondences between combinations and Pascal's triangle; communicate to others why the set of rational expressions is not a field.	STEM Readiness students at the above mastery level in mathematics: communicate patterns that occur in the coefficients and exponents in the expansion; explain correspondences between the set of rational expressions and the set of rational numbers.	STEM Readiness students at the mastery level in mathematics: solve problems using the binomial expansion; perform operations with rational expressions using the mathematics they know from the field properties of rational numbers.	STEM Readiness students at the partial mastery level in mathematics: expand binomials with coefficients of 1; multiply and divide rational expressions.	STEM Readiness students at the novice level in mathematics: create Pascal's triangle; add and subtract rational expressions.

Cluster	**Use polynomial identities to solve problems.**
Objectives	**Students will**
M.SRM.PRR.1	know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
Cluster	**Rewrite rational expressions**
Objectives	**Students will**
M.SRM.PRR.2	understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication and division by a nonzero rational expression; add, subtract, multiply and divide rational expressions.

Grade 11-12	**High School Mathematics STEM Readiness**
Standard	**Probability for decisions**

Performance Descriptors M.PD.SRM.PD				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
STEM Readiness students at the distinguished level in mathematics: model probability distributions in applied situations; construct an argument to support the appropriateness of a decision-making strategy based on expected value.	STEM Readiness students at the above mastery level in mathematics: analyze the shape of a probability distribution in terms of its data; compare or contrast decision-making strategies based upon expected value.	STEM Readiness students at the mastery level in mathematics: understand probability distributions and expected value as a mean; create a decision-making strategy based upon expected values.	STEM Readiness students at the partial mastery level in mathematics: calculate theoretical probabilities and graph their probability distributions; assign random variables to calculate an expected value.	STEM Readiness students at the novice level in mathematics: create basic sample spaces; given the random variable, calculate expected value.

Cluster	**Use probability to evaluate outcomes of decisions**
Objectives	**Students will**
M.SRM.PD.1	use probabilities to make fair decisions (e.g. drawing by lots, using a random number generator).
M.SRM.PD.2	analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Grade 11-12	**High School Mathematics STEM Readiness**
Standard	**Trigonometry of general triangles**

Performance Descriptors M.PD.SRM.TT				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
STEM Readiness students at the distinguished level in mathematics: justify and communicate to others the reason why some given measurements do not determine a unique triangle; justify to others the special unit circle values;	STEM Readiness students at the above mastery level in mathematics: identify real life applications that can be represented using triangles, decontextualize (represent symbolically) the situation, and solve using trigonometry; derive the special unit circle values;	STEM Readiness students at the mastery level in mathematics: derive and apply the appropriate formulas to calculate accurately and efficiently the unknown measurements of any triangle; use the radian measures of the unit circle in practical situations to explain how to	STEM Readiness students at the partial mastery level in mathematics: procedurally use trigonometric formulas to solve triangles; use the radian measures of the unit circle to calculate the arc length and	STEM Readiness students at the novice level in mathematics: use the definition of sine and cosine to solve right triangles; memorize the special unit circle values;

   determine an algorithm for finding accurate parameters of a trigonometric function that fits the data and justify their conclusions.	 choose appropriate tools to attend to the meaning of quantities that determine accurate parameters for the trigonometric function that fits the data.	calculate the arc length and determine trigonometric values; use technology to explore and choose trigonometric functions to model periodic data using accurate parameters.	determine trigonometric values; write the equation of a trigonometric function given the graph.	 Find the amplitude, period, and midline, given the graph of a trigonometric function.

Cluster	**Apply trigonometry to general triangles.**
Objectives	**Students will**
M.SRM.TT.1	derive the formula *A = 1/2 ab sin(C)* for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
M.SRM.TT.2	prove the Laws of Sines and Cosines and use them to solve problems.
M.SRM.TT.3	understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Grade 11-12	**High School Mathematics STEM Readiness**
Standard	**Functions and modeling**

Performance Descriptors M.PD.SRM.M

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
STEM Readiness students at the distinguished level in mathematics: utilize intuitive limits to analyze oblique asymptotes and continuity; model applications using composition of functions; model applications using exponential and logarithmic functions; contextualize and	STEM Readiness students at the above mastery level in mathematics: utilize intuitive limits to analyze horizontal asymptotes; recognize the relationships of domains and ranges in the functions; restrict the domain of a function to create an invertible function; contextualize a real world	STEM Readiness students at the mastery level in mathematics: utilize intuitive limits to analyze vertical asymptotes; make sense of composition relationships in problem situations; make sense of the inverse relationship between exponential and logarithmic functions; interpret the important	STEM Readiness students at the partial mastery level in mathematics: procedurally sketch polynomial and rational functions by hand; understand composition as a function of the result of another function; procedurally show that two functions are inverses by using composition; interpret rate of change of a	STEM Readiness students at the novice level in mathematics: procedurally graph polynomial and rational functions using technology; compute procedurally the composition of two functions; determine inverses of a function by interchanging x and y; interpret the key features of

decontextualize a real world situation given the key features of a function;	situation given a function;	features of a function in the context of the situation;	function;	a linear function;
contextualize a real world situation given different representation of a function;	decontextualize a real world situation given different representations of a function;	analyze different representations of a functions in the context of the situation;	analyze rate of change in different representations of a function;	analyze different representations of a linear function;
interpret adding a constant function to a given function in a real world situation;	find a pattern and communicate to others how the graph changes by adding a constant function;	build functions by adding a constant function to another function and search for regularity or trends;	build functions by adding a constant function to another function, where the constant is a positive integer;	identify a constant function;
interpret the composition of functions in a real world situation;	find a pattern and communicate to others how composition of functions can be used to do multi-step calculations;	build functions by using compositions and search for regularity or trends;	find the equation of a composition given a equations of the individual functions;	calculate values of composed functions;
use inverse relationships to explain the graphs of exponential and logarithmic functions;	use properties of logarithms to transform an expression;	use logarithms to represent exponential models and evaluate the logarithms using technology;	convert from an exponential equation to a logarithmic equation;	use technology to evaluate base 10 or base e logarithms;
justify and explain to others the symmetric properties of the trigonometric functions;	derive the special unit circle values;	make sense of the symmetry and periodicity of trigonometric functions using the unit circle;	calculate the sine, cosine, and tangent of special angles;	convert between degree and radian measure;
interpret periodic phenomena in terms of trigonometric functions;	extend methods of solving trigonometric equations using algebraic structure;	solve trigonometric equations in modeling contexts;	understand the relationship between trigonometric functions and their inverses;	using technology, calculate values of inverse trigonometric functions;
model periodic phenomena in terms of trigonometric functions involving phase shifts;	compare phase shifts in trigonometric functions to transformations of other functions;	calculate phase shift of a trigonometric function with accuracy and precision;	recognize the existence of phase shift, given the trigonometric function in algebraic form;	recognize the existence of phase shift, given the graph of a trigonometric function;

generalize one-to-one function properties to periodic trigonometric functions; interpret trigonometric inverse functions as means to simplify the solution of contextual problems; apply trigonometric identities to solve problems.	correlate domain restriction and horizontal line test to the creation of trigonometric inverses; apply trigonometric inverses to solve contextual problems; extend methods of verifying trigonometric identities using algebraic structure.	make sense of domain restriction for trigonometric functions with trigonometric inverses; solve trigonometric equations using trigonometric inverses; derive the addition and subtraction trigonometric identities.	recognize that trigonometric inverses have domain restrictions; understand when it is appropriate to use a trigonometric inverse; understand the structure of the addition and subtraction trigonometric identities.	understand that trigonometric functions have inverses; using technology, solve trigonometric equations; check the reasonableness of the addition and subtraction trigonometric identities by substitution of numerical values.

Cluster	**Analyze functions using different representations.**
Objectives	**Students will**
M.SRM.M.1	graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
M.SRM.M.2	graph rational functions, identifying zeros and asymptotes when suitable factorizations are available and showing end behavior.
M.SRM.M.3	graph exponential and logarithmic functions, showing intercepts and end behavior and trigonometric functions, showing period, midline, and amplitude.
Cluster	**Building a function that models a relationship between two quantities.**
Objectives	**Students will**
M.SRM.M.4	write a function that describes a relationship between two quantities.
M.SRM.M.5	compose functions. *For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.*
Cluster	**Build new functions from existing functions.**
Objectives	**Students will**
M.SRM.M.6	find inverse functions.
M.SRM.M.7	verify by composition that one function is the inverse of another.
M.SRM.M.8	read values of an inverse function from a graph or a table, given that the function has an inverse.
M.SRM.M.9	produce an invertible function from a non-invertible function by restricting the domain.
M.SRM.M.10	understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
Cluster	**Extend the domain of trigonometric functions using the unit circle.**
Objectives	**Students will**
M.SRM.M.11	use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.

M.SRM.M.12	use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
Cluster	**Model periodic phenomena using trigonometric functions.**
Objectives	**Students will**
M.SRM.M.13	understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
M.SRM.M.14	use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
Cluster	**Prove and apply trigonometric identities.**
Objectives	**Students will**
M.SRM.M.15	prove the addition and subtraction formulas for sine, cosine and tangent and use them to solve problems.

Advanced Mathematical Modeling

Students continue to build upon their algebra, and geometry foundations and expand their understanding through further mathematical experiences. The primary focal points of Advanced Mathematical Modeling include the analysis of information using statistical methods and probability, modeling change and mathematical relationships, mathematical decision making in finance, and spatial and geometric modeling for decision-making. Students learn to become critical consumers of the quantitative data that surround them every day, knowledgeable decision makers who use logical reasoning and mathematical thinkers who can use their quantitative skills to solve problems related to a wide range of situations. As they solve problems in various applied situations, they develop critical skills for success in college and careers, including investigation, research, collaboration and both written and oral communication of their work. As students work with these topics, they continually rely on mathematical processes, including problem-solving techniques, appropriate mathematical language and communication skills, connections within and outside mathematics and reasoning. Students also use multiple representations, technology, applications and modeling and numerical fluency in problem-solving contexts.

Developing college and career skills
The student develops and applies skills used in college and careers, including reasoning, planning and communication, to make decisions and solve problems in applied situations involving numerical reasoning, probability, statistical analysis, finance, mathematical selection and modeling with algebra, geometry, trigonometry and discrete mathematics.

Finance
The student creates and analyzes mathematical models to make decisions related to earning, investing, spending and borrowing money. Students investigate the purposes of various taxes and how they are calculated.

Probability
The student uses basic rules of counting and probability to analyze and evaluate risk and return in the context of everyday situations. Students continue to develop their understanding of probability concepts through experiments and simulations, using technology where appropriate.

Statistics
The student makes decisions based on understanding, analysis and critique of reported statistical information and summaries. Statistical methods are applied to design and conduct a study that addresses one or more particular questions. The student communicates the results of reported and student-generated statistical studies.

Modeling
The student analyzes numerical data in everyday situations using a variety of quantitative measures and numerical processes. Likewise, the student conducts investigations, models data, makes predictions and judges the validity of predictions based on data analysis. Mathematical models are used to represent, analyze and solve problems involving change.

Networks
The student uses a variety of network models represented graphically to organize data in quantitative situations, make informed decisions, and solve problems.

Social Decision Making
The student analyzes the mathematics behind various methods of ranking and selection and considers the advantages/disadvantages of each method.

Geometry
The student uses a variety of tools and methods to represent and solve problems involving static and dynamic situations.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 11-12	**Advanced Mathematical Modeling**			
Standard	**Developing college and career skills**			
Performance Descriptors M.PD.AMM.CCS				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Students at the distinguished level in Advanced Mathematical Modeling:	Students at the above mastery level in Advanced Mathematical Modeling:	Students at the mastery level in Advanced Mathematical Modeling:	Students at the partial mastery level in Advanced Mathematical Modeling:	Students at the novice level in Advanced Mathematical Modeling:
analyze given solution methods for errors and improvements and validate assessment with own methodology and independently present alternatives;	critique others solution methods and justify alternative strategies with logical arguments and present problem understanding and solution methodology to others;	collaborate with others and develop multiple strategies for problem solution and express personal understanding in writing;	plan and communicate (e.g. through writing or orally) strategies for problem solutions;	read problem statements and identify central issue and related questions;
synthesize source information, representations, and possible solution strategies, into a coherent mathematical package to create a reasoned solution.	analyze collected data set and determine if similar to previously considered situations.	re-examine collected information for hidden assumptions, relevant versus missing information, possible solution options, as well as source and accuracy of information.	represent collected information in terms of appropriate mathematical language. (For example: figures, equations, tables, graphs, and verbal descriptions)	find and collect data from appropriate sources related to a problem.
Cluster	**Math as a Language**			

Objectives	**Students will**
M.AMM.CCS.1	demonstrate reasoning skills in developing, explaining and justifying sound mathematical arguments and analyze the soundness of mathematical arguments of others.
M.AMM.CCS.2	communicate with and about mathematics orally and in writing as part of independent and collaborative work, including making accurate and clear presentations of solutions to problems.
Cluster	**Tools for Problem Solving**
Objectives	**Students will**
M.AMM.CCS.3	gather data, conduct investigations and apply mathematical concepts and models to solve problems in mathematics and other disciplines.

Grade 11-12	**Advanced Mathematical Modeling**
Standard	**Finance**

Performance Descriptors M.PD.AMM.FI

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Students at the distinguished level in Advanced Mathematical Modeling: calculate, consider and communicate precisely to others the advantages and disadvantages of various financial decisions; develop strategy to exceed budget expectation (for example, emergency expenditures), and incorporate changes into personal plan.	Students at the above mastery level in Advanced Mathematical Modeling: create and compare multiple representations of given financial situations and explain how they are related; discern discretionary from essential spending and incorporate plans for future large purchase into budget.	Students at the mastery level in Advanced Mathematical Modeling: create a mathematical model of a given financial situation and consider the impact of changing parameters; use personal data to create a realistic budget.	Students at the partial mastery level in Advanced Mathematical Modeling: evaluate mathematical models of given financial situations for various parameters; investigate cost of common expenses and represent in spreadsheet format for use in budget planning.	Students at the novice level in Advanced Mathematical Modeling: calculate financial values such as interest, loan payments, and taxes using given formulas; develop an outline for the components of a personal budget.

Cluster	**Understanding Financial Models**
Objectives	**Students will**
M.AMM.FI.1	determine, represent and analyze mathematical models for loan amortization and the effects of different payments and/or finance terms *For example: Auto, Mortgage, Credit Card.*
M.AMM.FI.2	determine, represent and analyze mathematical models for investments involving simple and compound interest with and without additional deposits *For example: Savings accounts, bonds, certificates of deposit.*
M.AMM.FI.3	determine, represent and analyze mathematical models for Inflation and the Consumer Price Index using concepts of rate of change and percentage growth.

Cluster	**Personal Use of Finance**
Objectives	**Students will**
M.AMM.FI.4	research and analyze personal budgets based on given parameters *For example: Fixed and discretionary expenses, insurance, gross vs. net pay, types of income (wage, salary, commission), career choice, geographic region, retirement and/or investment planning, etc.*
M.AMM.FI.5	research and analyze taxes including payroll, sales, personal property, real estate and income tax returns.

Grade 11-12	**Advanced Mathematical Modeling**			
Standard	**Probability**			
Performance Descriptors M.PD.AMM.P				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Students at the distinguished level in Advanced Mathematical Modeling: persevere in solving complex problem situations involving multiple counting and probability concepts and justify conclusions; alter behavior based upon risk predictions. Communicate decision making process to others.	Students at the above mastery level in Advanced Mathematical Modeling: communicate precisely to others the meaning of results including reasons for performing chosen calculations; make predictions related to the level of risk associated with a *personal* choice or activity by using probability.	Students at the mastery level in Advanced Mathematical Modeling: create coherent representations of problems and plan a solution pathway that incorporates proper use of probability and counting rules; make predictions related to the level of risk associated with a *teacher-assigned* activity by using probability.	Students at the partial mastery level in Advanced Mathematical Modeling: calculate compound and conditional probabilities; recognize situations where probability can be used to calculate assessments of risk, fairness and payoff.	Students at the novice level in Advanced Mathematical Modeling: calculate probabilities of simple events using appropriate counting principles; recognize situations where probability can be used to consider risk.

Cluster	**Analyzing Information Using Probability and Counting**
Objectives	**Students will**
M.AMM.P.1	use the Fundamental Counting Principle, Permutations and Combinations to determine all possible outcomes for an event; determine probability and odds of a simple event; explain the significance of the Law of Large Numbers.
M.AMM.P.2	determine and interpret conditional probabilities and probabilities of compound events by constructing and analyzing representations, including tree diagrams, Venn diagrams, two-way frequency tables and area models, to make decisions in problem situations.
Cluster	**Managing Uncertainty**
Objectives	**Students will**
M.AMM.P.3	use probabilities to make and justify decisions about risks in everyday life
M.AMM.P.4	calculate expected value to analyze mathematical fairness, payoff and risk.

Grade 11-12	**Advanced Mathematical Modeling**
Standard	**Statistics**

Performance Descriptors M.PD.AMM.S

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Students at the distinguished level in Advanced Mathematical Modeling:	Students at the above mastery level in Advanced Mathematical Modeling:	Students at the mastery level in Advanced Mathematical Modeling:	Students at the partial mastery level in Advanced Mathematical Modeling:	Students at the novice level in Advanced Mathematical Modeling:
consider unstated assumptions and additional influences and their impact on statistical conclusions, taking into account the context from which the data arose;	compare different studies to analyze the relative strengths and weaknesses of their conclusions;	listen to or read the arguments of others, critiquing the validity of the sampling procedure, type of study and conclusions drawn;	recognize flaws, bias or misuse of statistics;	identify information missing from media reports;
draw reasonable inferences from their study, recognizing possible reasons, like statistical bias, that limit generalizations of the result;	review results of their study and make plans for how it can be improved;	design, conduct and summarize the results of a statistical study;	distinguish between a population and sample; identify variable of interest and appropriate sampling technique;	create data displays and identify center, shape and spread;
present results of their study in multiple representations and can express the advantages of each.	generate and defend plausible arguments regarding the results of their study taking into account the context from which the data arose.	use tools and representations suitable to a specified audience to communicate precisely the results of their study in written, oral and graphical form.	make clear use of statistics vocabulary when discussing the results of their study.	describe the study conducted in terms appropriate to a general audience.

Cluster	**Critiquing Statistics**
Objectives	**Students will**
M.AMM.S.1	identify limitations or lack of information in studies reporting statistical information, especially when studies are reported in condensed form..
M.AMM.S.2	interpret and compare the results of polls, given a margin of error.
M.AMM.S.3	identify uses and misuses of statistical analyses in studies reporting statistics or using statistics to justify particular conclusions, including assertions of cause and effect versus correlation.
M.AMM.S.4	describe strengths and weaknesses of sampling techniques, data and graphical displays and interpretations of summary statistics and other results appearing in a study, including reports published in the media.

Cluster	**Conducting Statistical Analysis**
Objectives	**Students will**
M.AMM.S.5	identify the population of interest, select an appropriate sampling technique and collect data
M.AMM.S.6	identify the variables to be used in a study
M.AMM.S.7	determine possible sources of statistical bias in a study and how such bias may affect the ability to generalize the results
M.AMM.S.8	create data displays for given data sets to investigate, compare, and estimate center, shape, spread and unusual features
M.AMM.S.9	determine possible sources of variability of data, both those that can be controlled and those that cannot be controlled
Cluster	**Communicating Statistical Information**
Objectives	**Students will**
M.AMM.S.10	report results of statistical studies to a particular audience, including selecting an appropriate presentation format, creating graphical data displays and interpreting results in terms of the question studied.
M.AMM.S.11	communicate statistical results in both oral and written formats using appropriate statistical and nontechnical language.

Grade 11-12	**Advanced Mathematical Modeling**			
Standard	**Modeling**			
Performance Descriptors M.PD.AMM.M				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Students at the distinguished level in Advanced Mathematical Modeling: justify how large quantity matrix manipulations in a spreadsheet environment may result in incorrect answers as a result of sensitivity and error; recognize and utilize occurrences of a range of mathematical models in their own lives.	Students at the above mastery level in Advanced Mathematical Modeling: realize that large quantity matrix manipulations in a spreadsheet environment may result in incorrect answers as a result of sensitivity and error; use their model to make and justify predictions and communicate the rationale for choosing a specific model.	Students at the mastery level in Advanced Mathematical Modeling: use spreadsheets and matrices as tools for solution of problems involving large data sets and develop algebraic sense of matrix operations; convert between representations (verbal, table, graph or equation) and calculate appropriate modeling equations, choosing the best type of model.	Students at the partial mastery level in Advanced Mathematical Modeling: arrange data in spreadsheet or matrices to ready it for manipulation. Complete complex matrix operations; use appropriate technology to fit a specified type of modeling equation to a given data set.	Students at the novice level in Advanced Mathematical Modeling: equate arrays with tabular data forms. Develop familiarity with large data sets and complete simple matrix operations; create plots of data and/or examine a graph and determine whether or not a relationship exists.
Cluster	**Managing Numerical Data**			
Objectives	**Students will**			
M.AMM.M.1	solve problems involving large quantities that are not easily measured			

M.AMM.M.2	use arrays to efficiently manage large collections of data and add, subtract and multiply matrices to solve applied problems.
Cluster	**Modeling Data and Change with Functions**
Objectives	**Students will**
M.AMM.M.3	determine or analyze an appropriate model for problem situations - including linear, quadratic, power, exponential, logarithmic and logistic functions *For example: stopping distance, period of a pendulum, population growth, Richter Scale, Fujita Tornado Scale*
M.AMM.M.4	determine or analyze an appropriate cyclical model for problem situations that can be modeled with trigonometric functions*For example: predator-prey models, tide heights, diurnal cycle, music.*
M.AMM.M.5	determine or analyze an appropriate piecewise model for problem situations *For example: postal rates, phase change graphs, sales tax, utility usage rates.*
M.AMM.M.6	solve problems using recursion or iteration *For example: fractals, compound interest, population growth or decline, radioactive decay.*
M.AMM.M.7	collect numerical bivariate data; use the data to create a scatter plot; determine whether or not a relationship exists; if so, select a function to model the data, justify the selection and use the model to make predictions.

Grade 11-12	**Advanced Mathematical Modeling**
Standard	**Networks**

Performance Descriptors M.PD.AMM.N

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Students at the distinguished level in Advanced Mathematical Modeling: apply network skills to situations in their own lives.	Students at the above mastery level in Advanced Mathematical Modeling: make comparisons between types of networks and choose appropriate networks for a given task.	Students at the mastery level in Advanced Mathematical Modeling: construct and analyze networks to optimize scheduling of tasks.	Students at the partial mastery level in Advanced Mathematical Modeling: identify the type and features of given networks.	Students at the novice level in Advanced Mathematical Modeling: describe the order of tasks based on a given network or flow chart.

Cluster	**Networking for Decision Making**
Objectives	**Students will**
M.AMM.N.1	solve problems involving scheduling or routing situations that can be represented by a vertex-edge graph; find critical paths, Euler paths, Hamiltonian paths, and minimal spanning trees *For example: Konigsberg bridge problem, mail vs. Fed Ex delivery routes, kolam drawings of India, traveling salesman problem, map coloring*
M.AMM.N.2	construct, analyze, and interpret flow charts in order to develop and describe problem solving procedures

Grade 11-12	**Advanced Mathematical Modeling**
Standard	**Social Decision Making**

Performance Descriptors M.PD.AMM.SD

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Students at the	Students at the above	Students at the mastery	Students at the partial	Students at the novice level

distinguished level in Advanced Mathematical Modeling:	mastery level in Advanced Mathematical Modeling:	level in Advanced Mathematical Modeling:	mastery level in Advanced Mathematical Modeling:	in Advanced Mathematical Modeling:
consider implications of various ranking strategies on issues like boundaries and apportionment.	discuss fairness of various ranking strategies when applied to a given situation.	apply various ranking strategies to a given situation to determine the outcome using each strategy.	identify the type of ranking strategy that has been applied in a given situation.	use percentages and rounding to explain simple voting methods.

Cluster	**Making Decisions Using Ranking and Voting**
Objectives	**Students will**
M.AMM.SD.1	apply and analyze various ranking algorithms to determine an appropriate method for a given situation *For example: fair division, apportionment, search engine results.*
M.AMM.SD.2	analyze various voting and selection processes to determine an appropriate method for a given situation *For example: preferential vs. non-preferential methods, weighted voting.*

Grade 11-12	**Advanced Mathematical Modeling**			
Standard	**Geometry**			
Performance Descriptors M.PD.AMM.G				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Students at the distinguished level in Advanced Mathematical Modeling:	Students at the above mastery level in Advanced Mathematical Modeling:	Students at the mastery level in Advanced Mathematical Modeling:	Students at the partial mastery level in Advanced Mathematical Modeling:	Students at the novice level in Advanced Mathematical Modeling:
independently realize situational needs and apply proper techniques for problem solution;	equate understanding of specified problems to problems or situations with similar properties;	apply understanding to produce defensible solutions to specified problems;	experiment and self-discover how manipulatives can be used to generate conjectures;	utilize classroom manipulatives to solve problems;
apply transformations to create original works such as (but not limited to) animations, music compositions or project blueprints.	represent required transformations using matrices and implement the matrix representation using technology to solve complex problems.	identify and apply the required geometric transformation to model a given problem situation.	apply specific transformations to model well-explained problems.	understand the concept of vector and can perform matrix operations.

Cluster	**Concrete Geometric Representation (Physical Modeling)**
Objectives	**Students will**
M.AMM.G.1	create and use two- and three-dimensional representations of authentic situations using paper techniques or dynamic geometric environments for computer-aided design and other applications.

M.AMM.G.2	solve geometric problems involving inaccessible distances.
Cluster	**Abstract Geometric Representation (Matrix Modeling)**
Objectives	**Students will**
M.AMM.G.3	use vectors to represent and solve applied problems.
M.AMM.G.4	use matrices to represent geometric transformations and solve applied problems.

Transition Mathematics for Seniors

Transitional Math for Seniors prepares students for their entry-level credit-bearing liberal studies mathematics course at the post-secondary level. This course will solidify their quantitative literacy by enhancing numeracy and problem solving skills as they investigate and use the fundamental concepts of algebra, geometry, and introductory trigonometry.

These standards are grouped by concepts and are not necessarily arranged in any specific order for presentation.

Number and Quantity - The Real Number System
At this juncture in a student's mathematical experiences, they have been exposed to a wide variety of concepts that warrant revisiting. Equivalent representations of rational and irrational numbers represented by radical signs and rational exponents are investigated. Additional coverage is given to extending students' grasp of properties of exponents as they explore the similarities between manipulating integer and rational exponents.

Number and Quantity - The Complex Number System
Previously, students have examined the basic operations, equivalent representations and properties of the real number system. The predicament of taking even roots of a real number with a factor of negative one is now addressed in the exploration of the set of Complex Numbers. Basic operations, equivalent representations, properties and complex solutions to quadratic are addressed, as well as, the idea of conjugate pairs.

Algebra - Seeing Structure in Expressions
Students will deconstruct two and three term polynomials into equivalent expressions in factored form or by completing the square in order to identify zeros/roots/solutions along with maximum and/or minimum points.

Algebra - Arithmetic with Polynomials & Rational Expressions.
Students will review the arithmetic operations on polynomial and rational expressions. The emphasis here should be on creating equivalent expressions for both polynomial and rational expressions. We should emphasize the role factoring plays in terms of creating equivalent expressions relative to polynomials and rational expressions, as well as the role it plays in solving equations that contain these types of expressions.

Algebra - Creating Equations
Students will concentrate on creating equations or inequalities that model physical situations. The equations here should be in both one and two variables. Models should include linear, quadratic, rational, exponential and radical equations and inequalities. Systems of equations should also be included. An emphasis on the efficiency of solution as well as reasonableness of answers given physical limitations should be part of the discussion. The use of technology to model physical limitations is encouraged.

Algebra - Reasoning with Equations & Inequalities
Students will continue to develop and connect their mathematical understanding/knowledge of equations as they use and solve equations and inequalities in linear, rational, radical and quadratic formats.
Functions - Interpreting Functions
Students will continue to extend and develop their knowledge and understanding of functional notation and the concept of functions as they use, analyze, represent and interpret functions and their applications.

Functions - Building Functions
Students will further their knowledge of functions as they build new functions from existing functions.

Geometry – Congruence, Similarity, Right Triangles, & Trigonometry
In previous courses, students have had experience with rigid motions and have used these to develop notions about what it means for two objects to be congruent. Students also have established triangle congruence criteria, based on analyses of rigid motions and formal constructions. They have solved problems about triangles, quadrilaterals and other polygons. In addition students have employed the Pythagorean Theorem to solve right triangles directly and also by developing the Trig functions of sine, cosine and tangent. These objectives will serve to reinforce these ideas while deepening understanding through reasoning and proof.

Geometry - Expressing Geometric Properties with Equations
Building on their work with the Pythagorean Theorem to find distances, students will use a rectangular coordinate system to verify and prove geometric relationships and use formulas to solve related problems involving distance, perimeter and area.

Geometry - Modeling with Geometry
Students will apply various appropriate geometric concepts in contextual real world scenarios to solve design problems (which may involve topography, scale drawings, physical models, formulas or equations).

Statistics and Probability - Interpreting Categorical & Quantitative Data
Students will review the different ways of representing data. Both single variable data sets and two variable data sets should be presented. Statistical measures of central tendency, and variation should be discussed relative to one- variable data sets. Histograms, pie charts, bar graphs, box and whisker plots should be discussed. Scatter plots and linear regression should be covered relative to two variable data sets. The emphasis here should be how a change in the data influences the slope of the regression line.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 12	**Transition Mathematics for Seniors**			
Standard	**Number and Quantity - The Real Number System**			
Performance Descriptors M.PD.TMS.RN				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Transitions Mathematics students at the distinguished level in	Transitions Mathematics students at the above mastery level in	Transitions Mathematics students at the mastery level in mathematics:	Transitions Mathematics students at the partial mastery level in	Transitions Mathematics students at the novice level in mathematics:

mathematics: create and simplify expressions involving rational exponents and/or radicals; justify the operational properties of rational and irrational numbers.	mathematics: use a variety of tools to fluently simplify expressions involving rational exponents and/or radical; understand and apply relationships between rational and irrational numbers.	communicate how the properties of exponents justify procedures used to simplify radical expressions; given the domain determine the range for each of the operational properties of rational and irrational numbers.	mathematics: consider analogous problems in order to expand properties of integral exponents to include rational exponents ; perform operations using rational and irrational numbers.	convert between expressions containing rational exponents and expressions written in radical form; distinguish between rational and irrational numbers.

Cluster	**Extend the properties of exponents to rational exponents.**
Objectives	**Students will**
M.TMS.RN.1	explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $(5^{1/3})^3$ to be 5 because we want $(5^{1/3})^3 = 5^{1/3x3}$ to hold, so $(5^{1/3})^3$ must equal 5.
M.TMS.RN.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
Cluster	**Work with radicals and integer exponents.**
Objectives	**Students will**
M.TMS.RN.3	know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/(3^3) = 1/27$.

Grade 12	**Transition Mathematics for Seniors**			
Standard	**Number and Quantity - The Complex Number System**			
Performance Descriptors M.PD.TMS.CNS				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Transitions Mathematics students at the distinguished level in mathematics: analyze and communicate the flexible use of algebraic properties in performing arithmetic operations over the set of complex numbers; observe and communicate	Transitions Mathematics students at the above mastery level in mathematics: know and flexibly use algebraic properties to perform arithmetic operations over the set of complex numbers; graphically demonstrate the	Transitions Mathematics students at the mastery level in mathematics: use algebraic properties to perform arithmetic operations over the set of complex numbers; use appropriate methods to	Transitions Mathematics students at the partial mastery level in mathematics: recognize the need to extend the number system to include complex numbers; recognize when a quadratic	Transitions Mathematics students at the novice level in mathematics: recognize standard form of complex numbers; recognize that not all

that complex roots of quadratic equations appear in conjugate pairs.	implications of real versus complex roots of quadratic equations.	solve a quadratic equation that may have complex roots.	equation will have complex roots.	quadratic equations have real roots.

Cluster	**Perform arithmetic operations with complex numbers.**
Objectives	**Students will**
M.TMS.CNS.1	know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.
M.TMS.CNS.2	use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
M.TMS.CNS.3	find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
Cluster	**Use complex numbers in polynomial identities and equations.**
Objectives	**Students will**
M.TMS.CNS.4	solve quadratic equations with real coefficients that have complex solutions.

Grade 12	**Transition Mathematics for Seniors**			
Standard	**Algebra - Seeing Structure in Expressions**			
Performance Descriptors M.PD.TMS.SSE				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Transitions Mathematics students at the distinguished level in mathematics: make sense of quantities and explain their relationships in problem solving situations; fluently explain how to produce equivalent forms of quadratic expressions in order to identify and make sense of expression properties and routinely analyze and explain the merits of various pathways used to quadratic equations and analyze an applied problem modeled by a quadratic equation to communicate which	Transitions Mathematics students at the above mastery level in mathematics: fluently manipulate the form of algebraic expressions; routinely produce equivalent forms of quadratic expressions in order to identify and make sense of expression properties and solve quadratic equations in contextual situations and produce and interpret the concept of a maximum and minimum in a quadratic application.	Transitions Mathematics students at the mastery level in mathematics: deconstruct, identify and interpret parts of an algebraic expression in order to rewrite the expression; produce equivalent forms of quadratic expressions in order to identify and make sense of expression properties and plan, develop and apply a solution pathway to quadratic equations and apply the method of completing the square to find the maximum or minimum of a quadratic equation.	Transitions Mathematics students at the partial mastery level in mathematics: interpret algebraic expressions as being comprised of either single or multiple components; factor simple quadratic expressions with 1 as the coefficient of the quadratic term and solve quadratic equations and find the maximum or minimum of a quadratic equation in standard form using $\frac{-b}{2a}$.	Transitions Mathematics students at the novice level in mathematics: identify parts of algebraic expressions; recognize equivalent forms of quadratic expressions and write solutions of quadratic equations and identify whether a quadratic equation has a maximum or minimum.

coordinate of the maximum or minimum is the desired result for the problem.				

Cluster	**Interpret the structure of expressions.**
Objectives	**Students will**
M.TMS.SSE.1	use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
Cluster	**Write expressions in equivalent forms to solve problems.**
Objectives	**Students will**
M.TMS.SSE.2	choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. factor a quadratic expression to reveal the zeros of the function it defines. b. complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

Grade 12	**Transition Mathematics for Seniors**			
Standard	**Algebra - Arithmetic with Polynomials & Rational Expressions**			
Performance Descriptors M.PD.TMS.APR				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Transitions Mathematics students at the distinguished level in mathematics: develop an algorithm to communicate to others the general strategies; communicate to others why the set of polynomials is not a field; find polynomials that do not have zeros and explain why;	Transitions Mathematics students at the above mastery level in mathematics: use a variety of general methods to factor polynomials; identify important similarities between the set of polynomials and the set of integers; communicate to others why some polynomials do not have zeros;	Transitions Mathematics students at the mastery level in mathematics: factor nth degree polynomials into n factors looking for both general methods and shortcuts; perform operations with polynomials applying the mathematics they know from the field properties of integers; determine the factors of a polynomial from the zeros and vice versa; analyze this relationship to sketch the graph;	Transitions Mathematics students at the partial mastery level in mathematics: factor nth degree polynomials over the set of rational numbers; multiply polynomials; find the factors when given a graph;	Transitions Mathematics students at the novice level in mathematics: factor quadratic over the set of rational numbers; add and subtract polynomials with like denominators; find the zeros when given the graph;

communicate to others why the set of rational expressions is not a field; perform and justify operations on higher order polynomials.	explain correspondences between the set of rational expressions and the set of rational numbers; identify and use various tools to add, subtract and multiply polynomials.	perform operations with rational expressions using the mathematics they know from the field properties of rational numbers; use the integer system to analogously demonstrate that the polynomial system is closed with respect to addition, subtraction and multiplication.	add and subtract rational expressions; multiply linear and quadratic polynomials.	multiply and divide rational expressions; add and subtract linear and quadratic polynomials.

Cluster	**Perform arithmetic operations on polynomials.**
Objectives	**Students will**
M.TMS.APR.1	understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract and multiply polynomials.
Cluster	**Understand the relationship between zeros and factors of polynomials.**
Objectives	**Students will**
M.TMS.APR.2	identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
Cluster	**Rewrite rational expressions**
Objectives	**Students will**
M.TMS.APR.3	rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
M.TMS.APR.4	understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Grade 12	**Transition Mathematics for Seniors**
Standard	**Algebra - Creating Equations**
Performance Descriptors M.PD.TMS.ACE	

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Transitions Mathematics students at the distinguished level in mathematics: analyze and explain the reasonableness and	Transitions Mathematics students at the above mastery level in mathematics: continually evaluate the reasonableness of answers	Transitions Mathematics students at the mastery level in mathematics: create equations and inequalities to solve	Transitions Mathematics students at the partial mastery level in mathematics: manipulate formulas to isolate a particular variable;	Transitions Mathematics students at the novice level in mathematics: graph one variable equations; solve linear

efficiency of various solution processes; routinely analyze and explain the merits of various pathways used to quadratic equations or inequalities; use appropriate tools to analyze and compare solutions.	as they solve problems; solve quadratic equations and inequalities in contextual situations; explain the meaning of the solution to a system of equations.	problems; plan, develop and apply a solution pathway to quadratic equations and inequalities; solve systems of equations.	solve a quadratic equation by factoring; solve a system of linear equations algebraically.	equations and inequalities in one variable; solve a system of linear equations graphically.

Cluster	**Create equations that describe numbers or relationships.**
Objectives	**Students will**
M.TMS.ACE.1	create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions and simple rational and exponential functions.
M.TMS.ACE.2	create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
M.TMS.ACE.3	represent constraints by equations or inequalities and by systems of equations and/or inequalities and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

Grade 12	**Transition Mathematics for Seniors**			
Standard	**Algebra - Reasoning with Equations & Inequalities**			
Performance Descriptors M.PD.TMS.REI				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Transitions Mathematics students at the distinguished level in mathematics: analyze and explain the reasonableness and efficiency of various solution processes; routinely analyze and explain the merits of various pathways used to solve quadratic equations or inequalities; develop and defend conjectures as to when	Transitions Mathematics students at the above mastery level in mathematics: continually evaluate the reasonableness of answers as they solve problems; solve quadratic equations and inequalities in contextual situations; explain why some quadratic polynomials have complex	Transitions Mathematics students at the mastery level in mathematics: create equations and inequalities to solve problems; plan, develop and apply a solution pathway to quadratic equations that may have complex solutions; demonstrate that polynomial identities extend	Transitions Mathematics students at the partial mastery level in mathematics: manipulate formulas to isolate a particular variable; solve quadratic equations by factoring; solve quadratic equations;	Transitions Mathematics students at the novice level in mathematics: graph one variable equations; solve linear inequalities in one variable; recognize that the Fundamental Theorem of

quadratic polynomials will have complex solutions; use appropriate tools to analyze and compare solutions.	solutions; explain the meaning of the solution to a system of equations.	analogously to include the complex number system; solve systems of equations.	 solve a system of linear equations algebraically.	Algebra applies to quadratic polynomials; solve a system of linear equations graphically.

Cluster	**Understand solving equations as a process of reasoning and explain the reasoning.**
Objectives	**Students will**
M.TMS.REI.1	solve simple rational and radical equations in one variable and give examples showing how extraneous solutions may arise.
Cluster	**Solve equations and inequalities in one variable.**
Objectives	**Students will**
M.TMS.REI.2	solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
M.TMS.REI.3	solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*.
Cluster	**Solve systems of equations.**
Objectives	**Students will**
M.TMS.REI.4	solve systems of linear equations exactly and approximately (e.g. with graphs), focusing on pairs of linear equations in two variables
Cluster	**Represent and solve equations and inequalities graphically.**
Objectives	**Students will**
M.TMS.REI.5	explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Grade 12	**Transition Mathematics for Seniors**			
Standard	**Functions - Interpreting Functions**			
Performance Descriptors M.PD.TMS.IF				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Transitions Mathematics students at the distinguished level in mathematics: distinguish between relations that are and are not functions and	Transitions Mathematics students at the above mastery level in mathematics: connect and explain the relationship between a function and its graphical	Transitions Mathematics students at the mastery level in mathematics: demonstrate understanding of the meaning of function, including as it relates to	Transitions Mathematics students at the partial mastery level in mathematics: find the range, given a function (in function notation) and its domain;	Transitions Mathematics students at the novice level in mathematics: find an output, given a function and an input;

communicate reasoning;	representation;	sequences; contextually use and interpret statements written in function notation;		
use the contextual situation to make and justify predictions;	use key features of a function to describe its contextual situation;	interpret key features of a function in terms of context from any of its various representations;	identify key features of a function from any of its various representations;	identify slope and intercepts, given the graph of a linear function; identify intercepts, given the graph of an exponential function;
use functions to draw conclusions and further analyze relationships;	explain thought processes used in writing a function;	analyze the relationship between two quantities to write the function that models it;	determine whether a relationship is linear or exponential;	identify a linear relationship and write the function that models it;
find and compare the effectiveness of two plausible solution pathways;	explain thought processes used in writing a function in problem solving;	distinguish between the identifying features of linear and exponential functions; write a function, given any of its representations, to solve a problem;	identify key features needed to write a function from a graph or table;	identify a linear and an exponential function from a graph or a table;
communicate carefully formulated explanations of the parameters of a function and their relationship to the solution.	analyze meaning of the solution in the context of the problem.	interpret the contextual parameters of a function.	find contextual parameters of a linear function.	create a function to model a situation.

Cluster	**Understand the concept of a function and use function notation.**
Objectives	**Students will**
M.TMS.IF.1	understand a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
M.TMS.IF.2	use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context.
Cluster	**Interpret functions that arise in applications in terms of the context.**
Objectives	**Students will**
M.TMS.IF.3	for a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

Cluster	**Analyze functions using different representations.**
Objectives	**Students will**
M.TMS.IF.4	graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
M.TMS.IF.5	write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Grade 12	**Transition Mathematics for Seniors**			
Standard	**Functions - Building Functions**			
Performance Descriptors M.PD.				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Transitions Mathematics students at the distinguished level in mathematics:	Transitions Mathematics students at the above mastery level in mathematics:	Transitions Mathematics students at the mastery level in mathematics:	Transitions Mathematics students at the partial mastery level in mathematics:	Transitions Mathematics students at the novice level in mathematics:
justify to others the special unit circle values ;	derive the special unit circle values;	use the radian measures of the unit circle in practical situations to explain how to calculate the arc length and determine trigonometric values;	use the radian measures of the unit circle to calculate the arc length and determine trigonometric values;	memorize the special unit circle values;
collect data and construct models that can be described using translations and arithmetic combinations of function families;	determine how a quadratic or exponential model changes as components of the model change;	write a function describing the relationship between two quantities;	Use the recursive process to get a table or list of values of a function that represents a contextual relationship;	Use the recursive process to generate a table or list of the values of a given function;
apply the concept of a function and its inverse in contextual situations;	use appropriate tools to explore and analyze transformations of functions and inverses of functions;	determine the inverse of a function and the effect of various transformations of a function;	distinguish translations of functions based on their graphs;	given a table or set of ordered pairs representing a finite function, write the inverse of the function;
analyze and state the domain of the composition of two functions and how it	analyze and state the domain of the sum, difference, product, or	build a new functions by finding the sum, difference, product, quotient, or	build a new functions by finding the sum, difference, product, or quotient of other	build a new function by finding the sum, difference, or product of other

relates to the domains of the original functions.	quotient function as it relates to the domains of the original functions.	composition of other functions.	functions.	functions.

Cluster	**Build a function that models a relationship between two quantities.**
Objectives	**Students will**
M.TMS.BF.1	write a function that describes a relationship between two quantities. a. combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. b. compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.
Cluster	**Build new functions from existing functions.**
Objectives	**Students will**
M.TMS.BF.2	find inverse functions. a. solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = \frac{x+1}{x-1}$ for $x \neq 1$. b. verify by composition that one function is the inverse of another. c. read values of an inverse function from a graph or a table, given that the function has an inverse. d. produce an invertible function from a non-invertible function by restricting the domain.
Cluster	**Trigonometric Functions**
Objectives	**Students will**
M.TMS.BF.3	explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Grade 12	**Transition Mathematics for Seniors**			
Standard	**Geometry – Congruence, Similarity, Right Triangles, & Trigonometry**			
Performance Descriptors M.PD.TMS.CST				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Transitions Mathematics students at the distinguished level in mathematics: strategically use appropriate tools to demonstrate that SSA is not sufficient criteria for triangle congruence; routinely develop, challenge, explain and	Transitions Mathematics students at the above mastery level in mathematics: strategically use appropriate tools to create counterexamples to show that AAA does not determine triangle congruence; use	Transitions Mathematics students at the mastery level in mathematics: use transformations of rigid motion to develop and explain the definition of congruence; make formal geometric constructions with a variety of tools;	Transitions Mathematics students at the partial mastery level in mathematics: use appropriate tools to explore the requirements of triangle congruence ; make simple formal geometric constructions;	Transitions Mathematics students at the novice level in mathematics: recognize that geometric transformations of rigid figures will preserve congruence; identify corresponding parts of congruent figures and state

prove or disprove student conjectures;	appropriate tools to identify SAA or AAS as criteria for triangle congruence; formalize and defend how the steps in a construction result in the desired figure;			that they are congruent; perform simple constructions using paper folding, reflective devises, and dynamic geometric software;
identify and distinguish between correct reasoning and flawed reasoning; routinely develop, challenge, explain and prove or disprove student conjectures involving similarity relationships in geometric figures;	fluently analyze and prove geometric theorems; fluently analyze and prove geometric theorems involving similarity relationships in geometric figures;	understand and use stated assumptions, definitions, and previously established results in proving geometric theorems; understand and use stated assumptions, definitions, and previously established results in proving geometric theorems involving similarity in proving relationships in geometric figures;	use stated assumptions, definitions, and previously established results to informally justify geometric theorems; informally justify geometric theorems involving similarity and relationships in geometric figures;	logically order a given sequence of statements justifying geometric theorems; logically order a given sequence of statements to justify geometric theorems involving similarity;
create and explain a general process necessary to determine the point that will partition a segment of length d into segments whose lengths are in the ratio of a/b;	explain how to determine the point on a segment that will partition the segment in a given ratio;	determine the point on a segment that partitions the segment in a given ratio;	given a point on a segment determine the ratio of the partitions;	determine the midpoint of a segment;
develop contextual problems and explain the integral part right triangles play in the solutions; develop contextual problems and explain how the Pythagorean Identity would contribute to their solution.	apply right triangle solutions in contextual settings; analyze and explain why the Pythagorean Identity may or may not be useful in various contextual problems.	solve for missing parts of right triangles; prove the Pythagorean Identity and use it to determine values of other trigonometric functions.	solve for missing sides in right triangles; prove the Pythagorean Identity and use it to determine values of the sine or cosine functions.	solve for the hypotenuse in right triangles; use the Pythagorean Identity to determine values of the sine or cosine functions.

Cluster	**Prove geometric theorems**
Objectives	**Students will**

M.TMS.CST.1	prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
M.TMS.CST.2	prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
M.TMS.CST.3	prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other and conversely, rectangles are parallelograms with congruent diagonals.
M.TMS.CST. 4	make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment copying an angle, bisecting a segment, bisecting an angle, constructing perpendicular lines, including the perpendicular bisector of a line segment, and constructing a line parallel to a given line through a point not on the line.
Cluster	**Prove theorems involving similarity**
Objectives	**Students will**
M.TMS.CST.5	prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
M.TMS.CST.6	use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
Cluster	**Define trigonometric ratios and solve problems involving right triangles**
Objectives	**Students will**
M.TMS.CST.7	understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
M.TMS.CST.8	explain and use the relationship between the sine and cosine of complementary angles.
M.TMS.CST.9	use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Grade 12	**Transition Mathematics for Seniors**			
Standard	**Geometry - Expressing Geometric Properties with Equations**			
Performance Descriptors M.PD.TMS.GPE				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Transitions Mathematics students at the distinguished level in mathematics: give carefully formulated explanations showing how the distance formula is derived from the Pythagorean Theorem.	Transitions Mathematics students at the above mastery level in mathematics: create and explain examples in the coordinate plane that depict the connection between the distance formula and Pythagorean Theorem.	Transitions Mathematics students at the mastery level in mathematics: use geometric definitions and the coordinate plane to prove simple theorems and to solve related problems.	Transitions Mathematics students at the partial mastery level in mathematics: find the distance between two points on a coordinate plane by using the Pythagorean Theorem; repeat this process to find the perimeter of a polygon.	Transitions Mathematics students at the novice level in mathematics: use the Pythagorean Theorem to determine the length of a segment on a coordinate plane.

Cluster	**Use coordinates to prove simple geometric theorems algebraically**
Objectives	**Students will**
M.TMS.GPE.1	use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point (0, 2).
M.TMS.GPE.2	use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Grade 12	**Transition Mathematics for Seniors**			
Standard	**Geometry - Modeling with Geometry**			
Performance Descriptors M.PD.TMS.MG				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Transitions Mathematics students at the distinguished level in mathematics: classify three-dimensional objects by their cross-sections, communicate to others the reason that ratios and proportions can be used to find parts of similar figures.	Transitions Mathematics students at the above mastery level in mathematics: communicate the relationship between the area of the cross section and the volume of the object, communicate to others the relationships between the ratios of sides, areas and volumes of similar figures.	Transitions Mathematics students at the mastery level in mathematics: visualize relationships between cross-section and three-dimensional objects, use ratios and proportions of similar figures to solve real world problems.	Transitions Mathematics students at the partial mastery level in mathematics: describe the three-dimensional object by its cross-section, make a scale drawing on topographic grid paper.	Transitions Mathematics students at the novice level in mathematics: describe cross-section and three-dimensional objects, solve proportions.

Cluster	**Apply geometric concepts in modeling situations.**
Objectives	**Students will**
M.TMS.MG.1	apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with topographic grid systems based on ratios).

Grade 12	**Transition Mathematics for Seniors**			
Standard	**Statistics and Probability - Interpreting Categorical & Quantitative Data**			
Performance Descriptors M.PD.TMS.SPI				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Transitions Mathematics students at the distinguished level in mathematics:	Transitions Mathematics students at the above mastery level in mathematics:	Transitions Mathematics students at the mastery level in mathematics:	Transitions Mathematics students at the partial mastery level in mathematics:	Transitions Mathematics students at the novice level in mathematics:

analyze the validity of statistical summaries, analyze the validity of statistical summaries, predict and analyze the effect of a change in the data set.	justify the appropriateness of the selection of data displays and statistical measures, explain the interpretation of associations and trends, justify the appropriateness concerning the rate of change in a set of data.	create single-variable data displays and identify appropriate statistical measures to compare, summarize, and interpret data; create data displays for two variables and use them to describe associations and trends; interpret linear models in the context of the data.	create and compare data displays; create data displays for two variables and use them to recognize associations and trends; exhibit an informal understanding of rate of change in the data.	create data displays and find statistical measures; create data displays for two variables, use technology to determine the linear model.

Cluster	**Summarize, represent, and interpret data on two categorical and quantitative variables**
Objectives	**Students will**
M.TMS.SPI.1	represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Interpret linear models.
M.TMS.SPI.2	interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

FISCAL NOTE FOR PROPOSED RULES

Rule Title: **W. Va. 126CSR44BB, Policy 2520.2B Next Generation Content Standards and Objectives for Mathematics in West Virginia Schools**

Type of Rule: X Legislative ☐ Interpretive ☐ Procedural

Agency: West Virginia Department of Education

Address: Lou Maynus, Mathematics Coordinator
Office of Instruction
1900 Kanawha Boulevard, East
Building 6 Room 603
Charleston, WV 25305

Phone Number: 304-558-5325 Email: lmaynus@access.k12.wv.us

Fiscal Note Summary

Summarize in a clear and concise manner what impact this measure will have on costs and revenues of state government.

No costs or revenues will be impacted by the proposed amendment of W. Va. 126CSR44BB, Policy 2520.2B Next Generation Content Standards and Objectives for Mathematics in West Virginia Schools.

Fiscal Note Detail

Show over-all effect in Item 1 and 2 and, in Item 3, give an explanation of Breakdown by fiscal year, including long-range effect.

	FISCAL YEAR		
Effect of Proposal	Current Increase/Decrease (use"-")	Next Increase/Decrease (use"-")	Fiscal Year (Upon Full Implementation)
1. Estimated Total Cost	0	0	0
Personal Services	0	0	0
Current Expenses	0	0	0
Repairs & Alterations	0	0	0
Assets	0	0	0
Other	0	0	0
2. Estimated Total Revenues	0	0	0

Rule Title: **W. Va. 126CSR44BB, Policy 2520.2B Next Generation Content Standards and Objectives for Mathematics in West Virginia Schools**

3. Explanation of above estimates (including long-range effect);
Please include any increase or decrease in fees in your estimated total revenues.

No costs or revenues will be impacted by the proposed amendment of W. Va. 126CSR44BB, Policy 2520.2B Next Generation Content Standards and Objectives for Mathematics in West Virginia Schools.

MEMORANDUM

Please identify any areas of vagueness, technical defects, reasons the proposed rule **would not** have a fiscal impact, and/or any special issues **not** captured elsewhere on this form.

No costs or revenues will be impacted by the proposed amendment of W. Va. 126CSR44BB, Policy 2520.2B Next Generation Content Standards and Objectives for Mathematics in West Virginia Schools.

Signature of Agency Head or Authorized Representative — Date

[signature] 5-5-11

Action	Type
N: No Response	- Negative
NA: Not Accepted	+ Positive
A: Accepted	o Neutral

Date	Individual/Organization	Comments	Action/ Type	Rationale
5/17/11	Brenda Adkins	**§126-44BB-1. General.** Living in a world where we make things so complicated, wouldn't it be great if we would just simplify. While reading through all the language of the text, I started thinking how nice it would be for someone without a college degree in education to be able to read through the content and understand the items required for their children. Even this form, to fill out a response, is confusing.	N	
		§126-44B-2 Purpose I saw in the content standards, that finance should be taught. My question is where? As a system, we are failing our children. I see in some of the elementary math books, finance problems that the children can not relate to. What if we make it real for the students? Content which reflects making purchases of things they are interested in. Connections as to why mom and dad can not buy the latest toy because they have an electric bill to pay. Isn't math suppose to be about being real? Lets make it real so students can apply it to their life and start a basic understanding of economics. What is more real to the students, how to rotate a	NA	

		triange or figuring out how to afford their next computer game. We have high school seniors graduating that can not balance a checkbook but they can tell you the area of a circle. Let us get real with math so students can start applying the concepts to their life. How about requiring a finance class before graduation? For students that are not going into chemistry, engineering, or becoming a mathmatician, why do we require trig? I think we need to start becomming real, so our students can be better prepared for real life.		
5/19/11	Allan Meck	**§126-44B-4 Summary of the Content Standards and Objectives** In M.TMS.RN.1 (5^(1/3))^3 should be 5 not a root of 5. Also at least in the PDF form, this expression is ugly...very "fuzzy". I would hope we can fix that. In M.TMS.RN.3 1/(3^3) should be 1/27 not 127 Same ugly font issue in M.TMS.BF.2	A	Font issue corrected.
5/21/11	Charles Higginbotham	**§126-44B-4 Summary of the Content Standards and Objectives** WOW. That pretty much sums up my reaction to the high school section of the content standards. Again we are confusing the parents by our education. They still do not have any real idea to the terminology of Distinguished, Above Mastery and so forth. And now we are taking away the course titles of Algebra I, Geometry and the rest that	NA	The Next Generation Content Standards and Objectives focus primarily on the development of number sense at the elementary level. The changes at the K-8 level should assist high school teachers with the precursor skills needed to do high school mathematics.

		they can relate to. We need to see if the current 4 credit - Conceptual/Transitional works before we jump on the next bandwagon. As a recently retired Math teacher of 35 years and with a year of substituting in all levels of high school math, I have seen several things that need corrected. And they are things that I have seen for several years as a teacher of Geometry and Algebra II. Students must have some number sense before they enter the high school curriculum. they need to be able to add, subtract and multiply whole numbers with out a calculator. In Algebra II we waste 1/4 of the instructional year teaching how to solve simple one! and two step equations. You can not move on and expect your students to master the content if they can not do the simple things. I do not know what the real answer is but I don't think this reorganization is a move in the right direction.		
5/23/11	Grant Spencer	**§126-44B-4 Summary of the Content Standards and Objectives** My concern is that ALL of our students will not be able to adjust to such a wide scope of mathematics. There doesn't seem to be anywhere for the NON-MATH students to flourish at their own pace. Most students will struggle with making the connections between the different branches of high school math. Additionally, massive PD will be needed to update our teachers to become strong in all areas, instead of just Algebra 1 or just Geometry. Are students still going to be required to have 4 Math credits? The difficulty level of Math 2 , Math 3, and Math 4 is too intense for most students. What happens to AP Calculus and AP Statistics? I love Math. I understand that a lot of work was put in to this, especially from higher education. It seems to me that they have created a Math Utopia that could only be successful in a perfect world, with Math robots as the students. I think the reality is that this approach	N	Teaching discrete pieces of mathematical content is the reason our students are unable to make connections to the real world. Our teachers must prepare to help students make sense and reason about the mathematics.

		will create intense frustration among teachers and students that will ! result in weaker Math skills and less problem solvers.		
5/23/11	Kyle Berry	**§126-44BB-1. General.** I love the new standards. I would like to see specific exemplars that accompany the performance descriptors. I think teachers and students alike need to see examples of what those descriptors would look like. We don't tell the architect and engineer that we want a "tall building with lots of windows, plants in the lobby, and a parking garage", we give them specs and diagrams to make sure they have the right idea of what we want. I don't mean for every teacher and student to have the exact same expectation, but having a general blueprint for success would be very helpful in my opinion. I kept many of my excellent student work samples to show future classes what exemplary products have looked like before and encouraged them to put their best spin on it. Overall, though, I am very excited about the NxG standards.	N	
5/25/11	keith ross	**§126-44BB-1. General.** I am concerned that students will not be as able to succeed in undergraduate Math and Science fields. As an AP Chemistry and AP Physics teacher, I find that precalculus and calculus are the two most important classes that future scientists and engineers can take in high school. Successful AP calc teachers teach the content in a more engaging manner than the typical college instructor. The policy seems to take a one size fits all approach to	NA	Pre-Calculus and Calculus are still optional courses for our students.

		math education. Future engineers and scientists will be disadvantaged if the implementation process keeps them from precalc and calculus in high school. Private Prep schools have lost some of the advantage that they used to have because as more and more public high school students are successfully completing AP math and science classes. This policy threatens to give some of the advantage back to the private schools.		
6/2/11	Sue Hvizdos	**§126-44B-4 Summary of the Content Standards and Objectives** My worst fear with this proposed policy change is that higher education will fail to recogize Math I, Math II, etc as mathematics credits/electives. It was not too long ago that WVU failed to recognize CATS 10 as a laboratory class. With perseverance and a lot of student anxiety/tears, they finally agreed to acknowledge this course as a lab course. Needless to say, I am also concerned about out-of-state universities questioning the rigor and relevance of this sequence of courses. I am also concerned about student readiness for the ACT/SAT college entrance examinations. Will this sequence of courses (Math I, Math II, Math III) prepare our students for the rigors of a high stakes achievement test? I am not sure of the answer. I believe that the current CSOs (although they are not perfect) will adequately prepare our students for post-secondary educational studies. I would rather attempt to tweak the current CSOs then abandon them for the integrated series. I thank the committee members for having sacrificed	NA	The courses are recognized by Institutions of Higher Ed across the nation.

		their time and talents for producting the Next Generation CSOs. I ask that we continue to offer our top students the current curriculum and incorporate the integrated series for the below mastery students. Thanks for your allowing us to offer our input on this very important educaational issue.		
6/2/11	Gloria Burgess	**§126-44B-4 Summary of the Content Standards and Objectives** Please be specific and list prerequisites for each class. Our school currently allows students to take Alg II and Trig/Precalc at the same time, or Geometry and Alg II at the same time.	A	Prerequisites will be provided for the courses.
6/7/11	Tina M. Kirk	**§126-44BB-1. General.** There are 21st Century standards which we use now, and then there are the Next Generation standards which are not broken up by Algebra 1, Algebra 2, etc. Are these Next Generation standards supposed to eventually replace the 21st Century standards, or are they for a separate program, such as IMP math, which is broken up into levels 1, 2, 3, and 4?	A	IMP is a textbook. Our standards are written to ensure understanding of high school mathematics and to provide reasoning and sense making.
6/7/11	Audrey Boley	**§126-44BB-1. General.** I do ntt like this integrated format at the high school level. By the nature of high school math courses there must be scaffolding with prior knowledge without it being put in this format. This format failed with CATS in science and the University of Chicago school math project did not survive in WV schools. We have been trained in PLC's and CTN's to unwrap our current 21st century CSO's and we want to stay with that.	NA	The University of Chicago school math is a textbook curriculum. Our standards ensure students can reason and make sense with high school mathematics.

		This format is too chopped up to state on a college transcript that a student has completed the knowledge base expected in each traditional high school math course.		
6/9/11	Grant Spencer	**§126-44B-4 Summary of the Content Standards and Objectives** Questions/Concerns from Harrison County High School Math Meeting held on 6/9/11 1. Currently, our students have the opportunity to receive 8 hours of college math credit through WVU Math 126, Math 128, Math 153, and Math 154. Is there enough flexibility in the Next Generation sequence to allow these kids to continue receiving this opportunity? If there is a shift away from block scheduling and back to a 7 period day, these students will lose that opportunity. 2. This approach causes unique problems for high school teachers because of the complexity of the topics. Have you considered how this will work in a block schedule? The pace of instruction needed to accomplish this ambitious integration of topics will bury MOST students. 3. What will we do with teachers who are only certified through Algebra 1? Will middle school (5-8) teachers be able to teach the proposed Math 8 High School Math 1? 4. Will the counties have any flexibility in the implementation time schedule? We will potentially waste 3-5 years of the students' math education if grades 3-12 are done all at once. Can we write a waiver to phase in the common core gradually? If so, can we begin with a slower implementation? 5. Will there be training for teachers? Who will pay for it? When will it be done? Where will it be done? 6. ALL students will not be able to comprehend Math 2? Is there support for Math 2?	NA	College credit will continue to be available. Students will have several options as the Next Generation Standards plan allows for personalized learning. Integration of topics means ensuring students have an understanding of the mathematics and the purpose for learning the mathematics. Research shows when students know why they are studying a topic, they are more likely to attain mastery of that goal. Teachers will continue to teach the subjects they are certified to teach. WVDE has created an optional schedule for implementation that allows for a phase in period for each grade level. Training will be provided through our Teacher Leadership Institute and RESA based trainings throughout the school year. Students struggling in Math 2 may need additional support to master the content standards and objectives.

		Topics like quadratics, complex numbers, inverse functions, systems, and graphing all parent functions on top of the Geometry are going to frustrate the majority of our students. 7. What are we going to do with the students who are traditionally on the applied track? Can they survive Math 2? 8. Where are the technical readiness 1 and 2 objectives? Where can they be found? 9. What will we do with the assessment change in 2014? Can the testing be phased in with leeway in the cutoff scores until the students and teachers make the necessary adjustments? 10. Can we see the mapping of the old CSOs and the new CSOs? 11. We are concerned with Higher Education teacher training programs. Are they implementing it now so these students are ready to teach differently from the way they were taught?		Technical Readiness Objectives have not been created and are not included in this policy. An official crosswalk from the current CSOs to the Next Generation CSOs is available on the Teach 21 website. Higher Education has been involved from the beginning of our work in WV and mathematics education.
6/12/11	Kathy Marino	**§126-44B-4 Summary of the Content Standards and Objectives** 1. I am concerned how Math I and Math II will serve our Algebra Support students' needs. How will these students complete 50-some Math I CSO's and 60-some Math II CSO's? 2. Who will pay to train teachers? We newly adopted textbooks this year. Even if we go to online texts, there will still be costs. Would our county taxpayers would be expected to purchase new materials before this adoption period is complete? 3. Will tech readiness be part of the math curriculum in the high school or in the tech centers? 4. Science teachers throughout the state found that	NA	The Next Generation Standards are organized in clusters and accompanying objectives that provide very specific information to make clear what our children must know, understand and be able to do. Math I organizes six units divided into 21 clusters. These clusters are further described by the objectives, some of which are visited throughout the units. Math II is organized around six units with 28 clusters. Mathematics is integrated in K-8 and then again in Pre-calculus, Calculus and Probability and Statistics. The only courses that

		the integrated CATS9 and CATS10 science courses "watered" down the science curriculum. Are we headed down the same path with these integrated math courses? 5. Will vertical training be part of the training? All math teachers need to clearly understand the skills and concepts that will be taught in each class. How will you certify the 5-9 teacher that will now teach concepts beyond their certification? Hopefully not with a quick one week course. 6. Our young adults that are close to 30 years of age were put in a new reading program when they were 2nd graders. Many of them struggled in reading because they were part of the K-1 implementation. Are our students going to struggle if we don't implement Math I-IV separately? 7. Are we really "fixing" our "broken" system? Referring to WV as having a "broken" system was offensive to WV's mathematics teachers who work diligently to educate our math students.		have ever been separated were Algebra and Geometry. The difference in the traditional pathway and the Next Generation pathway is movement of 20% of content in courses to ensure students know the relationship between the symbolic manipulations in Algebra and Geometry.
6/13/11	Susan Alvis on behalf of Robert C Byrd High School Mathematics Department	**§ 126-44BB-1 General** June 10, 2011 Dear State Board of Education Member: We have reviewed the letter sent by Bridgeport High School, and while agreeing with its sentiment, we wish to point out the following inaccuracies and concerns. 1. The math teachers at RCB are certified math majors grades 7-12. 2. The Board of Education did not request input from us or the other four high schools. 3. The advanced math students will do well	N	

		with any curriculum. It is the lower level and average students who need our concern. These students need a revamped curriculum that emphasizes mastery and skills at the lower levels. 4. Current senior math will not change despite the addition of a Math IV course. 5. Technical Readiness I has no CSO's and should be taught at the technical center. 6. By 2014, Math I and Math II should be fully implemented at our school. We would request a one year waiver to implement Math III and Math IV. 7. The remaining four high schools in the county have a higher percentage of special education students and qualify for lower socio-economic programs. According to WVEIS on the Web's Needy Students Report: Bridgeport has 13% needy, Liberty has 49% needy, Lincoln 44% needy, RCB has 47% needy, and South Harrison has 38% needy. 8. We do not believe that Bridgeport High School's letter reflects our concerns. They did not attend the mandated common core standards meeting on Thursday, June 9, 2011. We respectfully ask the Board to consider the needs of all the high schools and their student in the Harrison County School System. We are officially and formally requesting that the Board endorse the implementations of the common core standards with a one year extension in Math III and Math IV. Sincerely, Leon Pilewski, RCB Principal Susan Alvis, RCB Mathematics Teacher and Department Chairman Paula Nelson, RCB Mathematics Teacher Mary Beth Paletta, RCB Mathematics Teacher Cathy Pizzino, RCB Mathematics Teacher		

		Chris Sprenger, RCB Mathematics Teacher **§126-44BB-2 Purpose** June 10, 2011 Dear State Board of Education Member: We have reviewed the letter sent by Bridgeport High School, and while agreeing with its sentiment, we wish to point out the following inaccuracies and concerns. 1. The math teachers at RCB are certified math majors grades 7-12. 2. The Board of Education did not request input from us or the other four high schools. 3. The advanced math students will do well with any curriculum. It is the lower level and average students who need our concern. These students need a revamped curriculum that emphasizes mastery and skills at the lower levels. 4. Current senior math will not change despite the addition of a Math IV course. 5. Technical Readiness I has no CSO's and should be taught at the technical center. 6. By 2014, Math I and Math II should be fully implemented at our school. We would request a one year waiver to implement Math III and Math IV. 7. The remaining four high schools in the county have a higher percentage of special education students and qualify for lower socio-economic programs. According to WVEIS on the Web's Needy Students Report: Bridgeport has 13% needy, Liberty has 49% needy, Lincoln 44% needy, RCB has 47% needy, and South Harrison has 38% needy. 8. We do not believe that Bridgeport High School's letter reflects our concerns. They did not attend the mandated common core standards		

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		the addition of a Math IV course. 5. Technical Readiness I has no CSO's and should be taught at the technical center. 6. By 2014, Math I and Math II should be fully implemented at our school. We would request a one year waiver to implement Math III and Math IV. 7. The remaining four high schools in the county have a higher percentage of special education students and qualify for lower socio-economic programs. According to WVEIS on the Web's Needy Students Report: Bridgeport has 13% needy, Liberty has 49% needy, Lincoln 44% needy, RCB has 47% needy, and South Harrison has 38% needy. 8. We do not believe that Bridgeport High School's letter reflects our concerns. They did not attend the mandated common core standards meeting on Thursday, June 9, 2011. We respectfully ask the Board to consider the needs of all the high schools and their student in the Harrison County School System. We are officially and formally requesting that the Board endorse the implementations of the common core standards with a one year extension in Math III and Math IV. Sincerely, Leon Pilewski, RCB Principal Susan Alvis, RCB Mathematics Teacher and Department Chairman Paula Nelson, RCB Mathematics Teacher Mary Beth Paletta, RCB Mathematics Teacher Cathy Pizzino, RCB Mathematics Teacher Chris Sprenger, RCB Mathematics Teacher **§ 126-44BB-4 Summary of the Content**		

Standards and Objectives

June 10, 2011

Dear State Board of Education Member:

We have reviewed the letter sent by Bridgeport High School, and while agreeing with its sentiment, we wish to point out the following inaccuracies and concerns.

1. The math teachers at RCB are certified math majors grades 7-12.

2. The Board of Education did not request input from us or the other four high schools.

3. The advanced math students will do well with any curriculum. It is the lower level and average students who need our concern. These students need a revamped curriculum that emphasizes mastery and skills at the lower levels.

4. Current senior math will not change despite the addition of a Math IV course.

5. Technical Readiness I has no CSO's and should be taught at the technical center.

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We respectfully ask the Board to consider the needs of all the high schools and their student in the

		Harrison County School System. We are officially and formally requesting that the Board endorse the implementations of the common core standards with a one year extension in Math III and Math IV. Sincerely, Leon Pilewski, RCB Principal Susan Alvis, RCB Mathematics Teacher and Department Chairman Paula Nelson, RCB Mathematics Teacher Mary Beth Paletta, RCB Mathematics Teacher Cathy Pizzino, RCB Mathematics Teacher Chris Sprenger, RCB Mathematics Teacher		
6/13/11	Susan Barrett	**§126-44BB-1 General** Susan Naylor is a coach. Shelly Prince not price The statements at the beginning of each grade level and the cluster titles make the NxG CSOs more coherent and easier to understand. **§126-44BB-4 Summary of the Content Standards and Objectives** M.PD.2.MD – • Mastery, third statement should say "within 100" rather than "with 100" • M.2.MD.2 is not captured in the performance descriptors at all. M.PD.2.G – • Mastery – last statement would be more clear if it said "recognize equal shares may have different shapes" rather than "recognize equal pieces may have different shapes"	A	Changes made to teacher's title and name. Changes made to improve these 2nd, 3rd and 4th grade performance descriptors.

		M.PD.3OA – • Above Mastery – third statement should say "explain strategies for recalling multiplication and division facts" rather than "explain strategies for recalling multiplication and division problems" • Last statement could be more closely aligned with objective if it said "use estimation to check reasonableness of answers" • Introducing use of a letter to represent an unknown and order of operations are not mentioned in the performance descriptors. These are new concepts in third grade that are expanded in subsequent grades. • Last statement in Above Mastery column is unclear; it does not require problem solving and seems less difficult than statements at lower levels. M.PD.3.MD – • Mastery – first statement needs a space between "of" and "liquid" • Statements in columns are not horizontally aligned • Above mastery statement for area is not clear M.PD.3.G – • Specifying the use of pattern blocks in the performance descriptors narrows the objective unnecessarily • Mastery - Putting a semicolon rather than "and" between "…groups overlap" and "model equal…" would separate these two distinct skills more clearly. M.PD.4.OA – • Novice – last statement should say "given" rather than "give" • Last statement under Above Mastery seems more challenging than statement under Distinguished. • Distinguished – first statement is very general – it sounds like the math practices but is not in any context related to the objectives that follow		

		• Novice – second statement "state multiplication facts" does not demonstrate understanding of the objectives listed • Mastery – finding all factor pairs of a number is not stated and probably should be; likewise with determining if a number is prime or composite. The statement under master could be moved to above mastery. • It is at fourth grade level that students are expected to demonstrate fluency and accuracy with addition and subtraction using the standard algorithms but this is not mentioned in the performance descriptors. • Operations with fractions in fourth grade are limited to certain cases; this is not apparent in reading the performance descriptors.		
6/9/11	Paul Howe	See attached letter.	NA	The Common Core Mathematics working group of 85 K-12 and 8 Higher Education Mathematics faculty chose to organize the high school mathematics content according to learning progressions. Advantages of these Next Generation content standards and objectives for high school mathematics are too many to list here but a few big ideas are: A powerful organizing principle of the Next Generation West Virginia State Standards is that of progressions, where an idea is reinforced over grade levels to build depth of understanding. For example, the Domain Number and Operations in Base 10 extends from K-2, with Grade K calling for

				students to work with numbers 11-19, Grade 1 focusing on 2 digit numbers, and Grade 2 extending the idea to 3 digit numbers. In the same way, the high school standards allow for progressions to be developed that relate to important ideas in algebra. For example, there is a Domain called Linear, Quadratic, and Exponential Models across Math I, II, III, with Math I focusing on linear and exponential functions, Math II extending the ideas to quadratic functions, and Math III incorporating logarithms. In contrast, the alternate track squeezes quadratic functions into Algebra I and students have a year hiatus from algebraic modeling during the Geometry course. We know how quickly ideas can degrade when they are not reinforced—just think of how much students lose over a summer-- so changing gears abruptly between courses has the potential to do real harm to retention. The Next Generation Standards also allow topics in geometry to benefit from being developed across courses. For example, continuing the modeling theme, Geometry is applied to modeling in Math III, keeping the geometric mode of thought alive. Proving geometric theorems is an

				important cluster in Math II, but it is presaged by an emphasis on transformations, congruence, and geometric constructions in Math I. Rigid motion in Math I is the starting point for developing the concept of geometric proof. The Next Generation Standards allow for much flexibility. All students will take Math I and II, and then choose mathematics courses that align to their personalized learning plan. Students will be permitted to take Math III STEM, Math III LA or the Technical Readiness mathematics courses. Flexibility has been built in to allow students to change their plans if they so choose through the addition of an optional STEM Readiness course. The optional Advanced Mathematical Modeling Course addresses financial literacy and other advanced modeling concepts needed for non-STEM majors. Some students will need to continue down the calculus pathway, however many students need more statistics and data analysis to make sense of the deluge of data and quantitative information in the media everyday. They need selected topics from matrices to understanding networks and counting techniques. They also need sophisticated

				knowledge of financial situations beyond basic consumer topics, including investment and debt. The Advanced Mathematical Modeling Course will address this need. Higher Education mathematics faculty has not only endorsed the courses but was part of the working group that created the additional optional courses. The mathematics content in Math I and II will prepare students for the mathematics needed for High School Chemistry while Math I, II and III will prepare students for the mathematics in Physics. Students on the accelerated pathway will be prepared mathematically to take Physics by the end of the tenth grade year. WV's previous Science CATS initiative was a state led effort. However, the Next Generation Mathematics Standards are organized following an international model after the highest performing countries in the world. These standards are supported by the National Council of Teachers of Mathematics, College Board and numerous other organizations. According to a new report by ACT, Inc *Affirming the Goal: Is College and Career Readiness an Internationally Competitive Standard?* We can be

				confident we are truly offering a world-class education. "In a global economy where all work can be digitized, automated, or outsourced anywhere on the globe-the benchmark for educational success is no longer measure by state standards, but by the best-performing educational systems internationally," said Andreas Schleicher, head of indicators and analysis division at OECD and director of PISA. Our teachers will need professional development to increase their skills and knowledge. The Next Generation Standards will guide teachers toward developing reasoning and sense making in high school mathematics. The Office of Instruction's tentative work plan includes RESA based professional development for high school teachers of mathematics over the next several years through full implementation of the standards. The partial quote referred to in the letter is from an email to a science teacher in Harrison county from Lou Maynus, WVDE Mathematics Coordinator is copied below for further clarification and to put the quote in the context intended. We use this quote from John Ewing (director of Math for America). He points out that the current status of

				high school mathematics is not a crisis but rather a long-term structural problem requiring fundamental long-term changes to address the changing needs of society and the world. **From:** Lou Maynus [mailto:lmaynus@access.k12.wv.us] **Sent:** Wednesday, May 25, 2011 8:26 AM **To:** 'Robin Anglin'; 'Cheryl McCarthy' **Cc:** 'Marty Burke (Marty Burke)' **Subject:** RE: math curriculum Hi Cheryl, This is an initiative of 44 states. The National Council of Teachers of Mathematics supports the organization of the coursework. Attached is a draft document that shows all the options our children will have. Notice, there is a lot of choice for our students. We plan to personalize learning more in mathematics education. This document is still in draft form and I have not been given official permission to distribute it, so please keep it to yourself for now. I have two children (one in 3rd

				grade and one in 5th) that will go through this entire sequence. This is an exciting time for them! For the first time, the standards explicitly require the teaching and learning with understanding. No more can we say we have taught mathematics by children being able to repeat an algorithm. They will understand what "completing the square" in Algebra I means. It really means to complete a square, but we have artificially separated the symbol manipulation from the visual geometry that helps to make sense of the algebra. We know our teachers are going to need support during this transition but with our current broken system in high school mathematics, this effort can only improve what is happening in our high school classrooms. I wish we had time to go through the material together to help you get a better sense of these wonderful improvements. However, please know as a parent of five (one of which is in high school right now). I am excited about the change. I hope to have documents to put on Teach 21 within the next few months that will help folks to get a better understanding of our common core state vision.

Lou Maynus, NBCT
Mathematics Coordinator
Office of Instruction

From: Robin Anglin [mailto:ranglin@access.k12.wv.us]
Sent: Wednesday, May 25, 2011 8:06 AM
To: 'Cheryl McCarthy'
Cc: 'Lou Maynus'; 'Marty Burke (Marty Burke)'
Subject: RE: math curriculum

Cheryl,

I am copying your email to Lou Maynus the Math Coordinator for our office. She will be able to explain the details and reason for the decision much better that I.

Robin Anglin
Science Coordinator
Office of Instruction

From: Cheryl McCarthy [mailto:cmccarth@access.k12.wv.us]
Sent: Tuesday, May 24, 2011 2:03 PM
To: ranglin@access.k12.wv.us
Subject: math curriculum

				As a science teacher, I am concerned about the possible change in the math curriculum to Math 1-4 in high school. To me it sounds a lot like the math version of CATS which I thought was a huge mistake, and since we no longer have CATS I guess that was also the view of the state. Do you know what is going on with the math changes or info that would make this seem like a good choice for our students? I am also extremely concerned since I have a 6th grade daughter that I expect to be in AP Calc her senior year, not math 4! Any insight would be appreciated. Thanks Cheryl McCarthy
6/9/11	Michael Queen	See attached letter	NA	See response to Paul Howe letter above.
6/9/11	Charles Reider	See attached letter	NA	See response to Paul Howe letter above.
6/9/11	Allen Gorrell	See attached letter	NA	See response to Paul Howe letter above.
6/9/11	David Sturm	See attached letter	NA	See response to Paul Howe letter above.

Action	
N	No Response
NA	Not Accepted
A	Accepted

Type	
-	Negative
+	Positive
o	Neutral

DATE	INDIVIDUAL ORGANIZATION	COMMENTS	ACTION/ TYPE	RATIONALE
§126-44B-1 General				
05-18	Patricia Coulter Math and Science Teacher coultergeist3@yahoo.com Clay County High School Clay County High School 1 Panther Drive Clay WV 25043	Looks good		
05-24	Jennifer Cole Teacher Harrison County Schools Bridgeport WV 26330	I am concerned with the possible changes to the CSO's. If we make these changes, it could eliminate the Alg. A and Alg. B, applied Geometry, and Conceptional Math classes that are offered to our students on a skilled path. I teach special education and several of my students are not going to college. What math classes will these students take in high school? I have concerns about this and need some clarification.		
	Stephanie Romanek math teacher sromanek@access.k12.wv.us	I am concerned as a teacher and a parent about how higher education will perceive the proposed math		

06-02	Ohio County Schools 1976 Park View Road Wheeling WV 26003	courses - High School Math I, High School Math II, etc. - on transcripts when students apply for college.		
06-02	Stephanie Romanek math teacher sromanek@access.k12.wv.us Ohio County Schools 1976 Park View Road Wheeling WV 26003	Some teachers are better than others in specific areas of mathematics. Having all teachers teach all content is a bad idea.		
		Questions/Concerns from Harrison County High School Math Meeting held on 6/9/11 1. Currently, our students have the opportunity to receive 8 hours of college math credit through WVU Math 126, Math 128, Math 153, and Math 154. Is there enough flexibility in the Next Generation sequence to allow these kids to continue receiving this opportunity? If there is a shift away from block scheduling and back to a 7 period day, these students will lose that opportunity. 2. This approach causes unique problems for high school teachers because of the complexity of the topics. Have you considered how this will work in a block schedule? The pace		

06-09	Heidi Griffith Curriculum Coordinator hgriffit@access.k12.wv.us Harrison County Schools 408 EB Saunders Way Clarksburg WV 26301	of instruction needed to accomplish this ambitious integration of topics will bury MOST students. 3. What will we do with teachers who are only certified through Algebra 1? Will middle school (5-8) teachers be able to teach the proposed Math 8 High School Math 1? 4. Will the counties have any flexibility in the implementation time schedule? We will potentially waste 3-5 years of the students' math education if grades 3-12 are done all at once. Can we write a waiver to phase in the common core gradually? If so, can we begin with a slower implementation? 5. Will there be training for teachers? Who will pay for it? When will it be done? Where will it be done? 6. ALL students will not be able to comprehend Math 2? Is there support for Math 2? Topics like quadratics, complex numbers, inverse functions, systems, and graphing all parent			

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§126-44B-2 Kindergarten Mathematics Content Standards and Objectives				
		I am excited to see these new objectives go into		

05-16	Janie Merendino Math Coach mmerendi@access.k12.wv.us Marion County schools 10 Diana dr Fairmont WV 26554	classrooms in WV! The negative national response to these are unfounded when those reports say the intent is to establish a one size fits all curriculum and to get a single source curriculum into all schools. The NXG objectives are truely WV's goals. There is one area I would like to see revised and that is the area of the performance descriptors. I worked on these for the 6th grade and as I have had time to reflect on other grade levels , now that our main task is completed- (and what a task that was but we had great leadership from Carla Williamson and Lou Maynus. In order to go deeper with these new objectives I believe the distinguished level descriptors need to ask for that. However as those are written distinguished is merely performing at the next grade levels objectives. I also believe this is due to the fact that elementary teachers wrote those and they ihave limited		

		they jhave limited knowledge of the deep mathematics due to their limit in content. WHy not tweak that distinguished descriptor to say things like Kindergarten students can solve and analyze problems , solve real world problems , test conjectures about numbers to 20, and create their own unique math problems using numbers to 20? These tasks require a deeper level of understanding of the concepts , not just more content. I would be happy to work on this task if needed. Check out the performance descriptors at distinguishe for 6th grade - my partner and I tried to avoid using the next grade levels objectives and tried to get at a deeper level of understanding. Thanks		
05-18	Patricia Coulter Math and Science Teacher coultergeist3@yahoo.com Clay County High School Clay County High School 1 Panther Drive Clay WV 25043	I appreciate the changes. It takes out a large part of the redundant information. Nice revision which makes the document more clear and concise. Thank you.		

06-02	Sue Hvizdos Departement Chairperson shvizdos@access.k12.wv.us Wheeling Park High School 1976 Park View Road Wheeling WV 26003	I would like to add the following changes to the Trigonometry CSOs: 1) I would delete M.O/T.3.10 (complex numbers in polar form) amd 2) I would delete M.O.T.3.11 (vectors). Both of these topics are covered sufficiently in physics and pre-calculus. With the removal of these CSOs, more time may be devoted to real-life applications of the remaining standards. I would also like more emphasis to be placed on the graphs of the trigonometric functions.		
§126-44B-2 Purpose				
05-18	Patricia Coulter Math and Science Teacher coultergeist3@yahoo.com Clay County High School Clay County High School 1 Panther Drive Clay WV 25043	Good		
	Valerie Helmstetter	The language developed for the stated descriptors provide a clear concise connection to student learning and knowledge. Intended learning, in my opinion, will		

05-23	Math Teacher vhelmste@access.k12.wv.us Poca High School Rte 2 Box 5B Poca WV 25159	be easier for the classroom to teacher to assess. Thus, both the student, parent and instructor will be provided with a stronger understanding of learning progress and measurement.		

RECEIVED
JUN - 9 2011
WV DEPT. OF EDUCATION
STATE SUPT'S OFFICE

June 6, 2011

Dr. Jorea M. Marple
WV Superintendent of Schools
West Virginia Department of Education
1900 Kanawha Boulevard East
Charleston, WV 25305

Dear Dr. Marple,

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Control # 11-643

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David M. Sturm

David Sturm, Member
Harrison County Board of Education

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Control #
11-642

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Allen Gorrell, Member
Harrison County Board of Education

June 6, 2011

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Control # 11-641

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Respectfully,

Charles Reider

Charles Reider, Vice-President
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Control # 11-640

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Control# 11-639

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- How will the Integrated Math Pathway be implemented? According to the WVDE website, kindergarten students will begin the Integrated Math Pathway in Fall 2011, 1^{st} grade Fall 2012, 2^{nd} grade Fall 2013, and 3^{rd} through 12^{th} in Fall 2014. This sequence will cause a nightmare for scheduling in secondary schools.

- Why are both 21^{st} Century Content Standards and the Integrated Math Pathway available for public comment at this time on the WVDE website?

- According to the WVDE website the comment period for the Integrated Math Pathway ends on June 13. The commoncore.org states that WV has already adopted the Integrated Math Pathway. We find that contradiction unsettling. Will our concerns and fears be addressed or has the decision already been made?

According to an email from Lou Maynus, NBCT, Mathematics Coordinator for West Virginia to BHS science teacher, Cheryl McCarthy, we (the WV Department of Education) need the Integrated Math Pathway implemented in our state to improve our "current broken system in high school mathematics." We are confident in the overall effectiveness of our current math program. Our test scores are exceptional, and our students are guided in the appropriate and individualized path to follow.

As a Harrison County Board of Education member I would like to formally state that not only is our system not broken, but our system works to the benefit of all students enrolled. We are requesting that the West Virginia Department of Education support our position and grant us permission to continue to follow the established 'traditional' pathway currently in use.

Respectfully,

Paul Howell, III, President
Harrison County Board of Education