WEST VIRGINIA SECRETARY OF STATE

NATALIE E. TENNANT

ADMINISTRATIVE LAW DIVISION

eFILED

9/12/2014 8:55:45 AM

OFFICE OF
WEST VIRGINIA SECRETARY OF STATE

FORM 5 -- NOTICE OF AGENCY ADOPTION OF A PROCEDURAL OR INTERPRETIVE RULE OR A LEGISLATIVE RULE EXEMPT FROM LEGISLATIVE REVIEW

AGENCY **Education**

RULE TYPE **Legislative Exempt** AMENDMENT TO EXISTING RULE **Yes** TITLE-SERIES **126-044B**

RULE NAME **21st Century Mathematics Content Standards and Objectives for West Virginia Schools (2520.2)**

CITE AUTHORITY **W. Va. Code §§29A-3B-1, et seq.: W. Va. Board of Education v. Hechler, 180 W. Va. 451; 376 S.E.2d 839 (1988)**

RULE IS LEGISLATIVE EXEMPT

Yes

CITE STATUTE(S) GRANTING EXEMPTION FROM LEGISLATIVE REVIEW

W. Va. Code §§29A-3B-1, et seq.; W. Va. Board of Education v. Hechler, 180 W. Va. 451; 376 S.E.2d 839 (1988)

THE ABOVE RULE IS HEREBY ADOPTED AND FILED WITH THE SECRETARY OF STATE. THE EFFECTIVE DATE OF THIS RULE IS

Tuesday, October 14, 2014

BY CHOOSING 'YES', I ATTEST THAT THE PREVIOUS STATEMENTS ARE TRUE AND CORRECT.

Yes

Charles K Heinlein -- By my signature, I certify that I am the person authorized to file legislative rules, in accordance with West Virginia Code §29A-3-11 and §39A-3-2.

Title-Series: 126-044B

Rule Id: 9560

Document: 26209

TITLE 126
LEGISLATIVE RULE
BOARD OF EDUCATION

SERIES 44B
21st CENTURY MATHEMATICS CONTENT STANDARDS AND OBJECTIVES FOR WEST VIRGINIA SCHOOLS (2520.2)

§126-44B-1. General.

1.1. Scope. -- W. Va. 126CSR42, West Virginia Board of Education Policy 2510, Assuring the Quality of Education: Regulations for Education Programs (hereinafter Policy 2510) provides a definition of a delivery system for, and an assessment and accountability system for, a thorough and efficient education for West Virginia public school students. Policy 2520.2 defines the content standards objectives for mathematics as required by Policy 2510.

1.2. Authority. -- W. Va. Constitution, Article XII, §2; W. Va. Code §18-2-5 and §18-9A-22.

1.3. Filing Date. -- September 12, 2014.

1.4. Effective Date. -- October 14, 2014.

1.5. Repeal of former rule. -- This legislative rule amends W. Va. 126CSR44B West Virginia Board of Education Policy 2520.2 "21st Century Mathematics Content Standards and Objectives for West Virginia Schools (2520.2)" filed June August 16, 2013 and effective September 16, 2013.

§126-44B-2. Purpose.

2.1. This policy defines the content standards and objectives for the program of study required by Policy 2510 in mathematics for grades nine through twelve.

§126-44B-3. Incorporation by Reference.

3.1. A copy of the 21st Century Mathematics Content Standards and Objectives for West Virginia Schools is attached and incorporated by reference into this policy. Copies may be obtained in the Office of the Secretary of State and in the West Virginia Department of Education, Office of Secondary Learning.

§126-44B-4. Summary of the Content Standards and Objectives.

4.1. The West Virginia Board of Education has the responsibility for establishing high quality standards pertaining to all educational standards pertaining to all education programs (W. Va. Code §18-9A-22). The content standards and objectives provide a focus for teachers to teach and students to learn those skills and competencies essential for future success in the workplace and further education. The document includes content standards for mathematics; an explanation of terms; objectives that reflect a rigorous and challenging curriculum; and performance descriptors.

§126-44B-5. Severability.

5.1. If any provision of this rule or the application thereof to any person or circumstance is held invalid, such invalidity shall not affect other provisions or applications of this rule.

Foreword

A 21st century mathematics curriculum is an increasingly important aspect of developing learners prepared for success in the 21st century. Thus, the West Virginia Board of Education and the West Virginia Department of Education are pleased to present Policy 2520.2, 21st Century Mathematics Content Standards and Objectives for West Virginia Schools. The West Virginia Mathematics Standards for 21st Century Learning includes 21st century *content* standards and objectives as well as 21st century standards and objectives for *learning skills* and *technology tools*. This broadened scope of mathematics curriculum is built on the firm belief that quality engaging instruction must be built on a curriculum that triangulates rigorous 21st century content, 21st century learning skills and the use of 21st century technology tools.

Committees of educators from across the state convened to revise the content standards and objectives. The overarching goal was to build a rigorous, relevant and challenging mathematics curriculum that would prepare students for the 21st century. West Virginia educators, including regular classroom teachers, special education teachers, and teachers representing higher education institutions played a key role in shaping the content standards to align with national standards, rigorous national assessments and research and best practice in the field of mathematics education. The contribution of these professionals was critical in creating a policy that is meaningful to classroom teachers and appears in a format that can easily be used and understood.

Policy 2520.2 is organized around the three major components of a standards-based curriculum: learning standards, instructional objectives and performance descriptors. The learning standards are the *broad descriptions* of what *all* students must know and be able to do at the conclusion of the instructional sequence. The accompanying grade-level objectives are specific descriptors of knowledge, skills and attitudes that when mastered will enable the student to attain the standard. The instructional objectives guide instructional *planning* and provide a basis for determining appropriate *assessments, instructional strategies and resources*. The performance descriptors provide the basis for *assessing* overall student competence of grade level standards. The performance descriptors define the five student performance levels ranging from novice to distinguished. With the ultimate goal of "learning for all," these descriptors allow the teacher, students and parents to judge the *level* of student proficiency in each 21st century learning standard.

In combination, the use of learning standards, instructional objectives and performance descriptors become a comprehensive guide for delivering a rigorous and relevant mathematics curriculum to all West Virginia students. These elements, when used to guide the instructional process and when delivered with the creativity and instructional expertise of West Virginia teachers, will become a powerful resource for preparing students to meet the challenges of the 21st century.

Explanation of Terms

Content Standards are broad descriptions of what students should know and be able to do in a content area. Content standards describe what students' knowledge and skills should be at the end of a 9-12 sequence of study.

Objectives are incremental steps toward accomplishment of content standards. Objectives are listed by grade level and are organized around the content standards. Objectives build across grade levels as students advance in their knowledge and skills.

Performance Descriptors describe in narrative format how students demonstrate achievement of the content standards. Line breaks within the narrative format indicate clusters of concepts and skills. West Virginia has designed five performance levels: distinguished, above mastery, mastery, partial mastery and novice. Performance Descriptors serve two functions. Instructionally, they give teachers more information about the level of knowledge and skills students need to acquire. Performance levels and descriptors are also used to categorize and explain student performance on statewide assessment instruments.

Distinguished: A student at this level has demonstrated exemplary performance. The work shows a distinctive and sophisticated application of knowledge and skills in real world situations that go beyond course or grade level applications.

Above Mastery: A student at this level has demonstrated effective performance and exceeds the standard. The work shows a thorough and effective application of knowledge and skills in real world situations within the subject matter and grade level.

Mastery: A student at this level has demonstrated competency over challenging subject matter, including knowledge and skills that are appropriate to the subject matter and grade level. The work is accurate, complete and addresses real world applications. The work shows solid academic performance at the course or grade level.

Partial Mastery: A student at this level has demonstrated limited knowledge and skills toward meeting the standard. The work shows basic but inconsistent application of knowledge and skills characterized by errors and/or omissions. Performance needs further development.

Novice: A student at this level has demonstrated minimal fundamental knowledge and skills needed to meet the standard. Performance at this level is fragmented and/or incomplete and needs considerable development.

Abbreviations

Content Area

M	Mathematics

High School Courses

Mathematics

A1	Algebra
A2	Algebra II
A3	Algebra III
C	Calculus
CM	Conceptual Mathematics
G	Geometry
PC	Pre-calculus
PS	Probability and Statistics
T	Trigonometry

Other Abbreviations

O	Objective
D	Performance Descriptors
S	Standard (Content Standard)

MATHEMATICS – POLICY 2520.2

The six principles for school mathematics, as articulated in *Principles and Standards for School Mathematics,* address six overarching themes to be considered when focused on the continuous improvement of mathematics education:

1. **Equity.** High expectations and strong support for all student
2. **Curriculum.** Coherent focus on important mathematics that is well-articulated across the grades
3. **Teaching.** Understanding what students know and need to learn and then challenging and supporting them to learn it well
4. **Learning.** Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge
5. **Assessment.** Assessment should support the learning of important mathematics and provide useful information to both teachers and students.
6. **Technology.** Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

The standards, objectives and performance descriptors presented in this policy are designed to provide clear, consistent priorities and focus, as well as depth of knowledge. The standards describe what each student of mathematics should be able to accomplish in grades 9-12. The objectives spiral upward through the grade levels, eliminating repetition of content and increasing in rigor and depth of knowledge throughout the student's academic career. It is important that all students value mathematics and see themselves as mathematical problem solvers who can communicate mathematically and make connections to other content areas and the real-world application of mathematics.

The vision of the West Virginia Board of Education and the West Virginia Department of Education includes the triangulation of mathematics content, learning skills and technology tools standards within each classroom so that students will be able to think critically, analyze information, comprehend new ideas, communicate, collaborate, solve problems and make decisions. All West Virginia mathematics teachers are responsible for the integration of Policy 2520.14 21st Century Learning Skills and Technology Tools in their classroom instruction.

It is important that teachers of mathematics become familiar with the performance descriptors at each grade level. The Mastery level performance descriptor, supported by the accompanying standard and objectives, describes student proficiency at that grade level. An understanding of the performance descriptors, standards and objectives provides a clear picture of what every student should know, understand and be able to do at each grade level. Teachers are encouraged to become familiar with the performance descriptors and

objectives at the previous and subsequent grade level to support a well-articulated curriculum. The abbreviation *e.g.* is used to indicate examples for teaching the objectives.

Policy 2510 states that "students in the professional pathway and college bound students in the skilled pathway, who do not achieve the State assessment college readiness benchmarks for mathematics, shall be required to take a college transition mathematics course during their senior year." In keeping with this policy, representatives from the West Virginia Department of Education and the Higher Education Policy commission assembled classroom teachers and professors of mathematics to establish the college readiness benchmarks for mathematics. An additional collaborative effort from classroom teachers and mathematics professors resulted in identification of a set of objectives from Policy 2520.2 courses in Algebra I, Geometry, Algebra II and Trigonometry that align to those benchmarks. The educational program for any student placed in a college transition mathematics course will be aligned to those objectives identified for Transition Mathematics. Therefore the college transition mathematics course is an individualized course relating to a student's identified skill deficiencies as related to previously approved objectives. Consequently, there is not an identified set of standards and objectives for the college transitions mathematics course required by Policy 2510.

Mathematics Content Standards 9-12

Standard 1: Number and Operations
Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will demonstrate an understanding of numbers, ways of representing numbers, and relationships among numbers and number systems, demonstrate meanings of operations and how they relate to one another, and compute fluently and make reasonable estimates.

Standard 2: Algebra
Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will demonstrate understanding of patterns, relations and functions, represent and analyze mathematical situations and structures using algebraic symbols, use mathematical models to represent and understand quantitative relationships, and analyze change in various contexts.

Standard 3: Geometry
Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships, specify locations and describe spatial relationships using coordinate geometry and other representational systems, apply transformations and use symmetry to analyze mathematical situations, and solve problems using visualization, spatial reasoning, and geometric modeling.

Standard 4: Measurement
Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will demonstrate understanding of measurable attributes of objects and the units, systems, and processes of measurement, and apply appropriate techniques, tools and formulas to determine measurements.

Standard 5: Data Analysis and Probability
Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them,
select and use appropriate statistical methods to analyze data, develop and evaluate inferences and predictions that are based on models, and apply and demonstrate an understanding of basic concepts of probability.

ALGEBRA I CONTENT STANDARDS AND OBJECTIVES

Algebra I objectives provide the gateway to all higher mathematics courses. An emphasis on conceptual development and multiple representations will be used to draw generalizations and to serve as a tool for solving real-world problems. Algeblocks may be used to bridge the gap from the concrete to the abstract. Available technology such as calculators, computers, and interactive utilities are to be used as tools to enhance learning. The West Virginia Standards for 21st Century Learning include the following components: 21st Century Content Standards and Objectives and 21st Century Learning Skills and Technology Tools. All West Virginia teachers are responsible for classroom instruction that integrates learning skills, technology tools and content standards and objectives.

Grade 9-12	**Mathematics: Algebra I**
Standard 2	Algebra
M.S.A1.2	Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will • demonstrate understanding of patterns, relations and functions, • represent and analyze mathematical situations and structures using algebraic symbols, • use mathematical models to represent and understand quantitative relationships, and • analyze change in various contexts.

Performance Descriptors (M.PD.A1.2)				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Algebra I students at the distinguished level will: formulate and simplify algebraic expressions for use in equations and inequalities, developing and justifying each step, derive and use the laws of integral exponents;	Algebra I students at the above mastery level will: formulate and simplify algebraic expressions for use in equations and inequalities, derive and use the laws of integral exponents; create, solve, and	Algebra I students at the mastery level will: formulate and simplify algebraic expressions for use in equations and inequalities, derive and use the laws of integral exponents; create, solve, and interpret solutions for	Algebra I students at the partial mastery level will: formulate and simplify algebraic expressions with integer coefficients for use in equations and inequalities, and use the laws of integral exponents;	Algebra I students at the novice level will: formulate and simplify algebraic expressions with whole number coefficients for use in equations and inequalities, and use integral exponents; create, solve, and interpret solutions for

create, solve, and concisely and clearly interpret solutions for multi-step equations; and solve literal equations; identify a real life situation with experiments to collect, organize, and analyze related data in a clear concise manner for display in multiple representations; formulate a conclusion; present the project with clarity and conciseness; model real-life situations involving exponential growth and decay equations and summarize the relationship in a clear, concise manner; develop and explain operations with and factoring of higher order polynomials, rational and radical	interpret solutions for multi-step equations; and solve literal equations; identify a real life situation and collect, organize, and analyze related data in a clear concise manner for display in multiple representations; formulate a conclusion; present the project; model real-life situations involving exponential growth and decay equations; develop and explain operations with and factoring of polynomials, rational and radical expressions. Use intercepts on a graph in problem	multi-step equations; and solve literal equations; identify a real life situation; collect, organize, and analyze related data for display in multiple representations; make a conclusion; present the project; describe real-life situations involving exponential growth and decay equations; develop and explain operations with and factoring of polynomials, rational and radical expressions;	create, solve, and interpret solutions for multi-step equations that contain only integral coefficients; and solve literal equations; identify a real life situation; collect and organize related data for display in multiple representations; make a conclusion; present the project; identify real-life situations involving exponential growth and decay equations; model and explain operations with and factoring of polynomials, rational and radical expressions;	multi-step equations that contain only whole number coefficients; and solve literal equations; identify a real life situation; collect and organize related data for display in multiple representations; make a conclusion; prove the existence of a pattern; identify real-life situations involving exponential growth; model operations with and factoring of polynomials, rational and radical expressions;

expressions. Use intercepts on a graph in problem solving; use simulations and rules of probability to design experiments to solve problems justifying the reasonableness of the approach in a clear, concise manner.	solving; use simulations and rules of probability to design and interpret experiments to solve problems.	use simulations and rules of probability to design experiments to solve problems.	use simulations and rules of probability to conduct and interpret experiments to solve problems.	use simulations and rules of probability to conduct experiments to solve problems.

Objectives	**Students will**
M.O.A1.2.1	formulate algebraic expressions for use in equations and inequalities that require planning to accurately model real-world problems.
M.O.A1.2.2	create and solve multi-step linear equations, absolute value equations, and linear inequalities in one variable, (with and without technology); apply skills toward solving practical problems such as distance, mixtures or motion and judge the reasonableness of solutions.
M.O.A1.2.3	evaluate data provided, given a real-world situation, select an appropriate literal equation and solve for a needed variable.
M.O.A1.2.4	develop and test hypotheses to derive the laws of exponents and use them to perform operations on expressions with integral exponents.
M.O.A1.2.5	analyze a given set of data and prove the existence of a pattern numerically, algebraically and graphically, write equations from the patterns and make inferences and predictions based on observing the pattern.
M.O.A.1.2.6	determine the slope of a line through a variety of strategies (e.g. given an equation or graph).
M.O.A1.2.7	analyze situations and solve problems by determining the equation of a line given a graph of a line, two points on the line, the slope and a point, or the slope and y intercept.
M.O.A1.2.8	identify a real life situation that involves a constant rate of change; pose a question; make a hypothesis as to the answer; develop, justify, and implement a method to collect, organize, and analyze related data; extend the nature of collected, discrete data to that of a continuous linear function that describes the known data set; generalize the results to make a conclusion; compare the hypothesis and the conclusion; present the project numerically, analytically, graphically and verbally using the predictive and analytic tools of algebra (with and without technology).

M.O.A1.2.9	create and solve systems of linear equations graphically and numerically using the elimination method and the substitution method, given a real-world situation.
M.O.A1.2.10	simplify and evaluate algebraic expressions • add and subtract polynomials • multiply and divide binomials by binomials or monomials
M.O.A1.2.11	create polynomials to represent and solve problems from real-world situations while focusing on symbolic and graphical patterns.
M.O.A1.2.12	use area models and graphical representations to develop and explain appropriate methods of factoring.
M.O.A1.2.13	simplify radical expressions • through adding, subtracting, multiplying and dividing • exact and approximate forms
M.O.A1.2.14	choose the most efficient method to solve quadratic equations by • graphing (with and without technology), • factoring • quadratic formula and draw reasonable conclusions about a situation being modeled.
M.O.A1.2.15	describe real life situations involving exponential growth and decay equations including $y=2^x$ and $y=(½)^x$; compare the equation with attributes of an associated table and graph to demonstrate an understanding of their interrelationship.
M.O.A1.2.16	simplify and evaluate rational expressions • add, subtract, multiply and divide • determine when an expression is undefined.
M.O.A1.2.17	perform a linear regression (with and without technology), • compare and evaluate methods of fitting lines to data. • identify the equation for the line of regression, • examine the correlation coefficient to determine how well the line fits the data • use the equation to predict specific values of a variable.
M.O.A1.2.18	compute and interpret the expected value of random variables in simple cases using simulations and rules of probability (with and without technology).
M.O.A1.2.19	gather data to create histograms, box plots, scatter plots and normal distribution curves and use them to draw and support conclusions about the data.
M.O.A1.2.20	design experiments to model and solve problems using the concepts of sample space and probability distribution.

M.O.A1.2.21	use multiple representations, such as words, graphs, tables of values and equations, to solve practical problems; describe advantages and disadvantages of the use of each representation.

Geometry Content Standards and Objectives

Geometry objectives are designed for students who have completed the objectives for Algebra I. Study includes experiences and activities that foster in students a feeling for the value of geometry in their lives. Emphasis is placed on development of conjectures by inductive processes using manipulatives and computer software. Cooperative learning groups are particularly effective in allowing students to become proficient in analyzing conjectures and in formulating both formal and informal proofs. Emphasis should be placed on connections to other branches of mathematics and other disciplines, and on workplace applications. The West Virginia Standards for 21st Century Learning include the following components: 21st Century Content Standards and Objectives and 21st Century Learning Skills and Technology Tools. All West Virginia teachers are responsible for classroom instruction that integrates learning skills, technology tools and content standards and objectives.

Grade 9-12	**Mathematics: Geometry and Applied Geometry**
Standard 3	Geometry
M.S.G.3	Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will • analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships, • specify locations and describe spatial relationships using coordinate geometry and other representational systems, • apply transformations and use symmetry to analyze mathematical situations, and • solve problems using visualization, spatial reasoning, and geometric modeling.

Performance Descriptors (M.PD.G.3)				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Geometry students at the distinguished level: investigate, create arguments, justify, compare and contrast, make conjectures, critique arguments and apply relationships involving the properties	Geometry students at the above mastery level: investigate, justify, make conjectures, compare and contrast, critique arguments and apply relationships involving the properties	Geometry students at the mastery level: investigate, make conjectures, compare and contrast, and/or apply relationships involving the properties of lines, polygons, measures of angles,	Geometry students at the partial mastery level: investigate and apply relationships involving the properties of lines, polygons, measures of angles, circles, Pythagorean Theorem,	Geometry students at the novice level: investigate relationships involving the properties of lines, polygons, measures of angles, circles, Pythagorean Theorem, transformational

of lines, polygons, measures of angles, circles, Pythagorean Theorem, transformational geometry, tessellating figures, trigonometric ratios, and the properties of Euclidean geometry with other geometries concepts of analytical geometry; construct the parts of a triangle and develop and justify logical concepts to be used in solving real-world problems; draw and justify conclusions in real-world settings and construct proofs, counterexamples, and logical arguments; identify a real life situation involving similarity; pose a question; make a hypothesis; collect, organize, and analyze	of lines, polygons, measures of angles, circles, Pythagorean Theorem, transformational geometry, tessellating figures, trigonometric ratios, the properties of Euclidean geometry with other geometries, and concepts of analytical geometry; construct the parts of a triangle and develop and justify logical concepts to be used in solving real-world problems; draw and justify conclusions in real-world settings and construct proofs, counterexamples, and logical arguments; identify a real life situation involving similarity; pose a question; make a hypothesis; collect,	circles, Pythagorean Theorem, transformational geometry, tessellating figures, trigonometric ratios, the properties of Euclidean geometry with other geometries, and concepts of analytical geometry; construct the parts of a triangle and develop logical concepts to be used in solving real-world problems; draw and justify conclusions in real-world settings and construct proofs and logical arguments; identify a real life situation involving similarity; pose a question; make a hypothesis; collect, organize, and analyze related data; make a conclusion; compare	transformational geometry, tessellating figures, trigonometric ratios, the properties of Euclidean geometry with other geometries, and concepts of analytical geometry; construct the parts of a triangle and use logical concepts to solve real-world problems; draw and justify conclusions in real-world settings and construct informal proofs; identify a real life situation involving similarity; pose a question; make a hypothesis; collect and organize data; make a conclusion; compare	geometry, tessellating figures, concepts of analytical geometry trigonometric ratios, and use analytical geometry to apply formulas; identify corresponding parts of similar triangles; draw conclusions in real-world settings and construct informal proof; identify a real life situation involving similarity; pose a question; make a hypothesis; collect and organize data; make a

related data; make a conclusion; compare the hypothesis and the conclusion; and present the project.	organize, and analyze related data; make a conclusion; compare the hypothesis and the conclusion; and present the project.	the hypothesis and the conclusion; and present the project.	the hypothesis and the conclusion.	conclusion; compare the hypothesis and the conclusion.

Objectives	**Students will**
M.O.G.3.1	represent geometric figures, such as points, lines, planes, segments, rays, and angles pictorially with proper identification and distinguish between undefined and defined terms.
M.O.G.3.2	differentiate and apply inductive and deductive reasoning, justify conclusions in real-world settings.
M.O.G.3.3	use the basic concepts of symbolic logic including identifying the converse, inverse, and contrapositive of a conditional statement and test the validity of conclusions with methods that include Venn Diagrams.
M.O.G.3.4	validate conclusions by constructing logical arguments using both formal and informal methods with direct and indirect reasoning.
M.O.G.3.5	construct formal and informal proofs by applying definitions, theorems, and postulates related to such topics as • complementary, • supplementary, • vertical angles, • angles formed by perpendicular lines, and justify the steps.
M.O.G.3.6	compare and contrast the relationships between angles formed by two lines cut by a transversal when lines are parallel and when they are not parallel, and use the results to develop concepts that will justify parallelism.
M.O.G.3.7	make conjectures and justify congruence relationships with an emphasis on triangles and employ these relationships to solve problems.
M.O.G.3.8	identify general properties of and compare and contrast the properties of convex and concave quadrilaterals • parallelograms • rectangles • rhombuses • squares • trapezoids
M.O.G.3.9	identify a real life situation that involves similarity in two or three dimensions; pose a question; make a hypothesis as to the answer, develop, justify, and implement a method to collect, organize, and analyze related data; generalize

	the results to make a conclusion; compare the hypothesis and the conclusion; present the project numerically, analytically, graphically and verbally using the predictive and analytic tools of algebra and geometry (with and without technology).
M.O.G.3.10	investigate measures of angles and lengths of segments to determine the existence of a triangle (triangle inequality) and to establish the relationship between the measures of the angles and the length of the sides (with and without technology).
M.O.G.3.11	verify and justify the basis for the trigonometric ratios by applying properties of similar triangles and use the results to find inaccessible heights and distances. Using the ratios of similar triangles to find unknown side lengths and angle measures, construct a physical model that illustrates the use of a scale drawing in a real-world situation.
M.O.G.3.12	apply the Pythagorean Theorem and its converse to solve real-world problems and derive the special right triangle relationships (i.e. 30-60-90, 45-45-90).
M.O.G.3.13	investigate measures of angles formed by chords, tangents, and secants of a circle and draw conclusions for the relationship to its arcs.
M.O.G.3.14	find angle measures of interior and exterior angles; given a polygon, find the length of sides from given data; and use properties of regular polygons to find any unknown measurements of sides or angles.
M.O.G.3.15	develop properties of tessellating figures and use those properties to tessellate the plane.
M.O.G.3.16	derive and justify formulas for area, perimeter, surface area, and volume using nets and apply them to solve real-world problems.
M.O.G.3.17	apply concepts of analytical geometry such as formulas for distance, slope, and midpoint and apply these to finding dimensions of polygons on the coordinate plane.
M.O.G.3.18	construct a triangle's medians, altitudes, angle and perpendicular bisectors using various methods; and develop logical concepts about their relationships to be used in solving real-world problems.
M.O.G.3.19	create and apply concepts using transformational geometry and laws of symmetry, of a • reflection, • translation, • rotation, • glide reflection, • dilation of a figure, and develop logical arguments for congruency and similarity.
M.O.G.3.20	compare and contrast Euclidean geometry to other geometries (i.e. spherical, elliptic) using various forms of communication such as development of physical models, oral or written reports.
M.O.G.3.21	approximate the area of irregularly shaped regions based on the approximations and the attributes of the related region, develop a formula for finding the area of irregularly shaped regions. Plan, organize and present results by

	justifying conclusions.

Algebra II Content Standards and Objectives

Algebra II objectives emphasize the use of investigation to more advanced functions, using them to solve real-world problems. Focus is on multiple representations to develop conjectures, testing and justifying validity. Calculators, computers, and interactive utilities are an integral part of instruction. The West Virginia Standards for 21st Century Learning include the following components: 21st Century Content Standards and Objectives and 21st Century Learning Skills and Technology Tools. All West Virginia teachers are responsible for classroom instruction that integrates learning skills, technology tools and content standards and objectives.

Grade 9-12	**Mathematics: Algebra II**
Standard 2	Algebra
M.S.A2.2	Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will • demonstrate understanding of patterns, relations and functions, • represent and analyze mathematical situations and structures using algebraic symbols, • use mathematical models to represent and understand quantitative relationships, and • analyze change in various contexts.

Performance Descriptors (M.PD.A2.2)

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Algebra II students at the distinguished level: develop and analyze practical situations to determine, graph and solve various types of equations, inequalities, and systems and express answers using various formats; extend the techniques of factoring	Algebra II students at the above mastery level: analyze practical situations to determine, graph and solve various types of equations, inequalities, and systems and express answers using various formats;	Algebra II students at the mastery level: determine, graph and solve various types of equations, inequalities, and systems and express answers using various formats; apply the appropriate method to factor	Algebra II students at partial mastery level: graph and solve various types of equations, inequalities, and systems and express answers using various formats; factor polynomials	Algebra II students at the novice level: graph and solve various types of equations, inequalities, and systems; factor most

polynomials and explain their application; convert between the graphs and equations of functions and conic sections using an analysis of their properties and graphing techniques and describe their characteristics; justify properties used to simplify and expand expressions and convert between appropriate forms; generate quadratic regressions to make predictions and present analysis of results; identify a real world situation that models quadratics, pose a question, collect and analyze data, and present and justify their results.	extend the techniques of factoring polynomials; convert between the graphs and equations of functions and conic sections using an analysis of their properties and graphing techniques and describe their characteristics; apply properties to simplify and expand expressions and convert between appropriate forms; generate quadratic regressions to make predictions and analyze results; identify a real world situation that models quadratics, pose a question, collect and analyze data, and present their results.	polynomials; convert between the graphs and equations of functions and conic sections using an analysis of their properties and graphing techniques; simplify and expand expressions and convert between appropriate forms; generate quadratic regressions to make predictions; identify a real world situation that models quadratics, pose a question, collect and analyze data.	when given the appropriate method; convert between the graphs and equations of functions and conic sections; simplify and expand expressions; make predictions given a quadratic regression; identify a real world situation that models quadratics and pose a question.	polynomials when given the appropriate method; graph functions and conic sections from the given equation; simplify and expand most expressions; recognize quadratic regressions; identify a real world situation that models quadratics.

Objectives	**Students will**
M.O.A2.2.1	determine equations of lines including parallel, perpendicular, vertical and horizontal lines, and compare and

	contrast the properties of these equations.
M.O.A2.2.2	factor higher order polynomials by applying various methods including factoring by grouping and the sum and difference of two cubes; analyze and describe the relationship between the factored form and the graphical representation.
M.O.A2.2.3	define complex numbers, simplify powers of 'i', perform basic operations with complex numbers, and give answers as complex numbers in simplest form.
M.O.A2.2. 4	simplify expressions involving radicals and fractional exponents, convert between the two forms, and solve equations containing radicals and exponents.
M.O.A2.2. 5	solve quadratic equations over the set of complex numbers: apply the techniques of factoring, completing the square, and the quadratic formula; use the discriminate to determine the number and nature of the roots; identify the maxima and minima; use words, graphs, tables, and equations to generate and analyze solutions to practical problems.
M.O.A2.2.6	develop and use the appropriate field properties of matrices by adding, subtracting, and multiplying; solve a system of linear equations using matrices; and apply skills toward solving practical problems.
M.O.A2.2.7	define a function and find its zeros; express the domain and range using interval notation; find the inverse of a function; find the value of a function for a given element in its domain; and perform basic operations on functions including composition of functions.
M.O.A2.2.8	analyze families of functions and their transformations; recognize linear, quadratic, radical, absolute value, step, piece-wise, and exponential functions; analyze connections among words, graphs, tables and equations when solving practical problems with and without technology.
M.O.A2.2.9	solve quadratic inequalities, graph their solution sets, and express solutions using interval notation.
M.O.A2.2.10	solve and graph the solution set of systems of linear inequalities in two variables by finding the maximum or minimum values of a function over the feasible region using linear programming techniques.
M.O.A2.2.11	solve practical problems involving direct, inverse and joint variation.
M.O.A2.2.12	analyze the conic sections; identify and sketch the graphs of a parabola, circle, ellipse, and hyperbola and convert between graphs and equations.
M.O.A2.2.13	solve absolute value inequalities graphically, numerically and algebraically and express the solution set in interval notation.
M.O.A2.2.14	define a logarithmic function, transform between exponential and logarithmic forms, and apply the basic properties of logarithms to simplify or expand an expression.
M.O.A2.2.15	identify a real life situation that exhibits characteristics of change that can be modeled by a quadratic equations; pose a questions; make a hypothesis as to the answer; develop, justify, and implement a method to collect, organize and analyze related data; extend the nature of collected, discrete data to that of a continuous function that describes

	the known data set; generalize the results to make a conclusion; compare the hypothesis and the conclusion; present the project numerically, analytically, graphically and verbally using the predictive and analytic tools of algebra (with and without technology).
M.O.A2.2.16	describe and illustrate how patterns and sequences are used to develop recursive and closed form equations; analyze and describe characteristics of each form.

Conceptual Mathematics Content Standards and Objectives

Conceptual Mathematics objectives include major topics from algebra and geometry and extend these ideas to practical usage. Basic ideas of probability and statistics and the mathematics of finance are included. These big ideas are to be presented in the context of their historical development. Full integration of calculators, computers, and interactive utilities are essential for mastery. The West Virginia Standards for 21st Century Learning include the following components: 21st Century Content Standards and Objectives and 21st Century Learning Skills and Technology Tools. All West Virginia teachers are responsible for classroom instruction that integrates learning skills, technology tools and content standards and objectives.

Grade 9-12	**Mathematics: Conceptual Mathematics**
Standard 2	Algebra
M.S.CM.2	Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will • demonstrate understanding of patterns, relations and functions, • represent and analyze mathematical situations and structures using algebraic symbols, • use mathematical models to represent and understand quantitative relationships, and • analyze change in various contexts.

Performance Descriptors (M.PD.CM.2)

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Conceptual Mathematics students at the distinguished level: research, create, apply, and compare a variety of problem-solving strategies to solve real-world problems and justify the reasonableness of the solutions; develop and solve	Conceptual Mathematics students at the above mastery level: create, apply, and compare a variety of problem-solving strategies to solve real-world problems and justify the reasonableness of the solutions;	Conceptual Mathematics students at the mastery level: apply and compare a variety of problem-solving strategies to solve real-world problems and justify the reasonableness of the solutions; solve application	Conceptual Mathematics students at the partial mastery level: apply problem-solving strategies to solve real-world problems; solve application	Conceptual Mathematics students at the novice level: apply problem-solving strategies to solve problems; recognize problems involving functions;

applications problems involving functions and assess their usefulness in the real world; pose questions, make hypotheses, and implement appropriate methods to collect, organize, analyze, draw conclusions, and present data (with and without technology); research and investigate real-world personal finance situations and differentiate between the types of personal finance functions in order to solve and make recommendations to optimize the situation.	develop and solve application problems involving functions and interpret and analyze their graphs; pose questions, make hypotheses, and implement appropriate methods to collect, organize, analyze, draw conclusions, and present data (with and without technology); research and investigate real-world personal finance situations and differentiate between the types of personal finance functions in order to solve the problem.	problems involving functions and interpret and analyze the graphs; pose questions, make hypotheses, and implement appropriate methods to collect, organize, analyze, draw conclusions, and present data (with and without technology); differentiate between the types of personal finance functions in order to solve real-world problems.	problems involving functions and read the graphs; implement appropriate methods to collect, organize, and present data (with and without technology); solve real-world problems when given the appropriate personal finance function.	collect, organize, and present data (with and without technology); compute using personal finance formulas.

Objectives	**Students will**
M.O.CM.2.1	use a variety of problem solving strategies (e.g., draw a diagram, look for a pattern, work backwards) to solve real-world problems.
M.O.CM.2.2	interpret graphs of functions including linear, quadratic, and exponential.
M.O.CM.2.3	solve application problems using linear, quadratic and exponential functions with emphasis on data collection and analysis.
M.O.CM.2.4	choose the appropriate formulas to solve workplace problems and judge the reasonableness of the solutions.
M.O.CM.2.5	describe and illustrate how calculating costs, simple and compound interest, finance charge, loan payment and tax functions are used to solve real-world problems.

M.O.CM.2.6	identify a real life situation that involves investing money over time; pose a question; make a hypothesis as to the answer; develop, justify, and implement a method to collect, organize, and analyze related data; generalize the results to make a conclusion; compare the hypothesis and the conclusion; present the project numerically, analytically, graphically and verbally using words, graphs, models, or tables (with and without technology).

Grade 9-12	**Mathematics: Conceptual Mathematics**
Standard 3	Geometry
M.S.CM.3	Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will • analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships, • specify locations and describe spatial relationships using coordinate geometry and other representational systems, • apply transformations and use symmetry to analyze mathematical situations, and • solve problems using visualization, spatial reasoning, and geometric modeling.

Performance Descriptors (M.PD.CM.3)

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Conceptual Mathematics students at the distinguished level: design and implement a project which applies concepts of geometry to compute measures and analyze connections between geometric shapes and their real-world applications.	Conceptual Mathematics students at the above mastery level: determine and apply concepts of geometry to compute measures and analyze connections between geometric shapes and their real-world applications.	Conceptual Mathematics students at the mastery level: apply concepts of geometry to compute measures and analyze connections between geometric shapes and their real-world applications.	Conceptual Mathematics students at the partial mastery level: use concepts of geometry to compute measures and model connections between geometric shapes and their real-world applications.	Conceptual Mathematics students at the novice level: recognize concepts of geometry to compute measures and describe connections between geometric shapes and their real-world applications.

Objectives	**Students will**
M.O.CM.3.1	apply concepts of geometry including the Pythagorean Theorem, similar triangles, and right triangle trigonometry.
M.O.CM.3.2	compute measures to solve real-world problems, using relationships involving perimeter, area, surface area and volume of geometric figures.

M.O.CM.3.3	analyze the connections of various geometric shapes and patterns to art, architecture, and nature.

Grade 9-12	**Mathematics: Conceptual Mathematics**			
Standard 3	**Data Analysis and Probability**			
M.S.CM.5	Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will • formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them, • select and use appropriate statistical methods to analyze data, • develop and evaluate inferences and predictions that are based on models, and • apply and demonstrate an understanding of basic concepts of probability.			
Performance Descriptors (M.PD CM.5)				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Conceptual Mathematics students at distinguished level: research and relate mathematical content to its historical development and connect to other disciplines; design and conduct probability investigations using counting techniques, and determine, analyze and communicate the results and develop	Conceptual Mathematics students at above mastery level: research and relate mathematical content to its historical development and integrate other disciplines into the study of mathematics; design and conduct probability investigations using counting techniques, and determine, analyze and communicate the results and develop	Conceptual Mathematics students at mastery level: relate mathematical content to its historical development and integrate other disciplines into the study of mathematics; design and conduct probability investigations using counting techniques, and determine, analyze and communicate the results;	Conceptual Mathematics students at partial mastery level: recognize mathematical content as it relates to its historical development and relate how other disciplines are integrated into the study of mathematics; conduct probability investigations using counting techniques, and determine, analyze and communicate the results;	Conceptual Mathematics students at novice level: recognize that mathematical content is related to its historical development and see how other disciplines are integrated into the study of mathematics; conduct probability investigations using counting techniques and communicate results;

rules of probability; compare and contrast more than one set of data that they collect, summarize, and interpret numerically and graphically.	rules of probability; collect, summarize, and interpret data numerically and graphically to make predictions.	collect, summarize, and interpret data numerically and graphically to make predictions.	collect and summarize data numerically and graphically.	collect and summarize data numerically and graphically.

Objectives	**Students will**
M.O.CM.5.1	relate mathematical content to its historical development.
M.O.CM.5.2	integrate other disciplines into the study of mathematics through simulations, research, and projects.
M.O.CM.5.3	determine possible outcomes using tree diagrams and the counting principles of permutations and combinations, develop conclusions and offer solutions for new situations, using real-world data.
M.O.CM.5.4	design and conduct probability investigations and then determine, analyze, and communicate the results.
M.O.CM.5.5	collect and interpret data using various methods of displaying numerical data, including frequency distributions, graphs, histograms, stem-and-leaf plots, and box-and-whiskers plots, using technology when appropriate.
M.O.CM.5.6	relate the measures of central tendency and the measures of dispersion to a normal distribution.
M.O.CM.5.7	apply the measures of central tendency and the measures of dispersion to workplace situations.
M.O.CM.5.8	use statistical tools for workplace applications such as quality control, marketing and predicting trends.

Algebra III Content Standards and Objectives

Algebra III is intended for students who have mastered the concepts of Algebra I, Geometry, and Algebra II. Algebra III objectives develop and extend properties of higher degree polynomial functions, rational functions, exponential functions and logarithmic functions using the common concepts and language of algebraic, graphical, and tabular representations. The use of analytic geometry for sense making, conceptual understanding of abstract ideas and modeling real world applications is stressed, making use of calculators, computers, and interactive activities. The West Virginia Standards for 21st Century Learning include the following components: 21st Century Content Standards and Objectives and 21st Century Learning Skills and Technology Tools. All West Virginia teachers are responsible for classroom instruction that integrates learning skills, technology tools and content standards and objectives.

Grade 9-12	**Mathematics: Algebra III**
Standard 2	**Algebra**
M.S.A3.2	Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will • demonstrate understanding of patterns, relations and functions, • represent and analyze mathematical situations and structures using algebraic symbols, • use mathematical models to represent and understand quantitative relationships, and • analyze change in various contexts.

Performance Descriptors (M.PD.A3.2)

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Algebra III students at the distinguished level: research practical solutions to choose appropriate representations from the families of functions using characteristics of the functions;	Algebra III students at the above mastery level: analyze practical solutions to compare and apply multiple representations of families of functions using characteristics of the functions;	Algebra III students at the mastery level: compare and apply multiple representations of families of functions using characteristics of the functions; recognize and	Algebra III students at the partial mastery level: explore multiple representations of families of functions using characteristics of the function;	Algebra III students at the novice level: recognize multiple representations of families of functions using basic characteristics of the functions;

demonstrate, relate, and assess connections between functions and their inverses, justify restricting the domain to guarantee an inverse, and apply transformations, compositions, and operations;	recognize, demonstrate, and relate connections between functions and their inverses, appropriately restrict the domain to guarantee an inverse and apply transformations, compositions, and operations;	demonstrate connections between functions and their inverses and apply transformations, compositions and operations;	recognize connections between functions and their inverses by performing transformations, compositions, and operations;	recognize inverse functions and perform compositions and arithmetic operations;
use properties of analytic geometry to apply elements of equations, interpret rates of change, convert between forms of equations, develop and justify the use of the distance and midpoint formulas;	use properties of analytic geometry to determine equations, interpret rates of change, convert between forms of equations, and develop the distance and midpoint formulas;	use properties of analytic geometry to determine equations, their components and relationships and apply the distance and midpoint formulas;	use properties from analytic geometry to determine slope, equations of circles, and apply the distance and midpoint formulas;	recognize slope of a line, equations of circles, and calculate distance and midpoint using formulas;
collaborate to choose a real world problem that can be modeled using algebraic and graphical techniques, predict, justify, and explain the model, and screen for extraneous solutions explaining their existence.	create models of real world problems using algebraic and graphical techniques, screen for extraneous solutions, and explain their existence.	model real world problems using algebraic and graphical techniques and screen for extraneous solutions.	solve real world problems using algebraic and graphing techniques and recognize extraneous solutions.	confirm solutions of real world problems using algebraic and graphical techniques and recognize extraneous roots.

Objectives	Students will
M.O.A3.2.1	use properties of analytic geometry to justify and use the distance and midpoint formulas and negative reciprocal criterion for non-vertical perpendicular lines.
M.O.A3.2.2	factor higher order polynomials by using techniques that can be applied to the factoring of second degree polynomials; relate factored forms of polynomials to graphs, tables, and solutions to problems in context.
M.O.A3.2.3	relate analytical attributes such as characteristics of zeros, x- and y- intercepts, symmetry, asymptotes, end behavior, maximum and minimum points, and domain and range, to graphical and algebraic representations of polynomials and rational functions.
M.O.A3.2.4	analyze the discriminant to classify the roots of quadratic equations with real coefficients, and relate the existence of x-intercepts of the graph to information obtained from the discriminant.
M.O.A3.2.5	solve equations with extraneous roots; explain why the extraneous roots are excluded from the solution set.
M.O.A3.2.6	compare and contrast the use of interval notation, set notation, and number line representations to express the domain and range of functions.
M.O.A3.2.7	compare and contrast the domain and range of a modeling function with the restricted domain and range used in a real world situation; justify the restricted domain and range choice for a problem in context.
M.O.A3.2.8	differentiate between functions and relations; evaluate, add, subtract, multiply, divide, rationalize, simplify, and compose functions (including rational, radical and those with fractional exponents); express domain and range of functions.
M.O.A3.2.9	convert between graphs and equations of circles identifying important features from either representation; translate from general form to standard form by completing the square and describe readily usable characteristics of each form; represent a circle as two functions graphically and algebraically.
M.O.A3.2.10	analyze a piecewise defined function in multiple representations, to give its domain, intercepts, range, constituent pieces as elementary functions, and end behavior; apply to real world data.
M.O.A3.2.11	determine the average rate of change of a function between any two points on its graph and use this rate to find the equation of a secant line; interpret the average rate of change to solve real world problems; relate signs of average rate of change to the function increasing or decreasing; and demonstrate a geometrical and conceptual understanding of the difference quotient.
M.O.A3.2.12	use synthetic division to divide a polynomial, verify a factor, and determine its roots; compare and contrast synthetic division to long division.
M.O.A3.2.13	investigate how the multiplicity of zeros of polynomial functions affects the graph; characterize a polynomial given the zeros, the behavior of the graph at the zeros, and the end-behavior.
M.O.A3.2.14	given the characteristics of a transformation involving polynomial, radical, absolute value, logarithmic, or exponential functions, determine a representative function; unravel the effect of a series of transformations using

	multiple representations.
M.O.A3.2.15	define and discuss one-to-one functions including the role of the Vertical and Horizontal Line Tests; use multiple representations in describing the relationship between a function and its inverse, including the domain and range of each; identify and explain the need for appropriate restrictions necessary to guarantee an inverse function; discuss the symmetrical relationship associated with the line y=x between the function and its inverse and explain the geometric reason the symmetry exists; demonstrate how to algebraically verify that two functions are inverses of each other.
M.O.A3.2.16	prioritize relevant techniques to graph a given rational function, explaining the relevance of symmetry, end behavior, and domain and range; use zeros of the denominator to differentiate between vertical asymptotes and points of discontinuity; use long division to determine end behavior and explain the role of quotient and remainder in the process; explain how the factors of the numerator and denominator can be used to analytically and graphically determine where the graph will fall above or below the x-axis.
M.O.A3.2.17	restrict the possible rational zeros of a polynomial function by using the Rational Zeros Theorem and Descartes' Rule of Signs; confirm the real zeros of a polynomial function by using the Remainder and Factor Theorems; approximate zeros of a polynomial or rational function using a graphing utility and the Intermediate Value Theorem.
M.O.A3.2.18	analyze polynomial equations with real coefficients and complex roots using factoring, the Conjugate Roots Theorem, the quadratic formula, or root restricting theorems; confirm roots using numerical and graphical methods; discuss and justify how the graph of a polynomial function gives information about complex zeros.
M.O.A3.2.19	compare and contrast the cases when $0<a<1$ and $a>1$ for the general exponential function $f(x) = a^x$: graphs, asymptotes, domain and range, and transformations. Interpret the number e as a limit and use e to build exponential functions modeling real world applications.
M.O.A3.2.20	use common and natural logarithms in the evaluation of logarithmic functions whose base is neither 10 nor e. Incorporate the change of base formula and properties of logarithms to simplify and expand algebraic expressions and to solve logarithmic and exponential equations.
M.O.A3.2.21	through algebraic, graphical, numerical, and verbal techniques, solve equations involving radical, exponential, and logarithmic expressions. Formulate strategies to solve real life problems including compound interest and exponential growth and decay.
M.O.A3.2.22	build on the skills of solving linear equations in two variables using elimination, substitution, or matrix methods to solve systems with three or more unknowns involving real world applications. Categorize systems of equations as zero, one, or infinitely many solutions, by both geometric and algebraic methods.

M.O.A3.2.23	work in groups to choose a real life situation that could be modeled by a polynomial, rational, exponential, or logarithmic function, and make a hypothesis, design an experiment, gather data, analyze data, refine the hypothesis into an appropriate mathematical model, use the model to make a prediction, test the prediction using the experimental setup, and compare the results. Present the collaboration as a project using words, graphs, tables, equations, and appropriate presentation tools.

Trigonometry Content Standards and Objectives

Trigonometry objectives emphasize making connections between right triangle trigonometry and circular functions. Calculators, computers, and interactive utilities will be used to enhance student learning. The West Virginia Standards for 21st Century Learning include the following components: 21st Century Content Standards and Objectives and 21st Century Learning Skills and Technology Tools. All West Virginia teachers are responsible for classroom instruction that integrates learning skills, technology tools and content standards and objectives.

Grade 9-12	**Mathematics: Trigonometry**
Standard 3	Geometry
M.S.T.3	Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will • analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships, • specify locations and describe spatial relationships using coordinate geometry and other representational systems, • apply transformations and use symmetry to analyze mathematical situations, and • solve problems using visualization, spatial reasoning, and geometric modeling.

Performance Descriptors (M.PD.T.3)				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Trigonometry students at the distinguished level: define and relate the six trigonometric functions in right triangles and in circular functions; graph and evaluate them and their inverse functions; solve trigonometric equations and apply them to real-	Trigonometry students at the above mastery level: define and relate the six trigonometric functions in right triangles and in circular functions; graph and evaluate them and their inverse functions; solve trigonometric equations and apply them to real-	Trigonometry students at the mastery level: define and relate the six trigonometric functions in right triangles and in circular functions; graph and evaluate them and their inverse functions; solve trigonometric equations and apply them to real-world problems;	Trigonometry students at the partial mastery level: define and relate the six trigonometric functions in right triangles and in circular functions; graph and evaluate them and their inverse functions; solve trigonometric equations;	Trigonometry students at the novice level: recognize the six trigonometric functions in right triangles and in circular functions, recognize the graphs and evaluate the functions and their inverses; solve trigonometric equations;

world problems; convert from degrees to radians (and vice versa) and test hypothesis or hypotheses to derive formulas to find applications of radian measure; determine the appropriate use for the Law of Sines and the Law of Cosines; apply to real-world situations; solve triangles and figures of multiple shapes; perform graphical and algebraic addition of vectors; convert complex numbers to polar form and graph in the polar coordinate plane and compare the graph to real world situations; identify three-dimensional vectors and use graphs, tables and equations to model periodic data sets and to analyze real	world problems; convert from degrees to radians (and vice versa) and test hypothesis to derive formulas to find applications of radian measure; determine the appropriate use for the Law of Sines and the Law of Cosines and solve triangles and figures of multiple shapes; perform graphical and algebraic addition of vectors; convert complex numbers to polar form and graph in the polar coordinate plane; compare the graph to real-world situations;	convert from degrees to radians (and vice versa) and develop formulas to find applications of radian measure; determine the appropriate use for the Law of Sines and the Law of Cosines and solve triangles; perform graphical and algebraic addition of vectors; convert complex numbers to polar form and graph in the polar coordinate plane;	convert from degrees to radians and develop formulas to find applications of radian measure; use for the Law of Sines and the Law of Cosines and solve triangles; perform graphical and algebraic addition of vectors; convert complex numbers to polar form and identify graphs in the polar coordinate plane;	recognize radians and formulas to convert from degrees to radians and recognize formulas to find applications of radian measure; recognize the Law of Sines and the Law of Cosines and find the area of triangles; recognize graphical and algebraic addition of vectors; recognize the conversion of complex numbers to polar form and identify graphs in the polar coordinate plane;

world problems; verify the basic identities and use them to verify and evaluate other identities.	verify the basic identities and use them to verify and evaluate other identities.	verify the basic identities and use them to verify other identities.	identify the basic identities and use them to verify other identities.	recognize the formulas for the basic identities.

Objectives	**Students will**
M.O.T.3.1	apply the right triangle definition of the six trigonometric functions of an angle to determine the values of the function values of an angle in standard position given a point on the terminal side of the angle. • determine the value of the other trigonometric functions given the value of one of the trigonometric functions and verify these values with technology. • using geometric principles and the Pythagorean Theorem, determine the six function values for the special angles and the quadrantal angles and use them in real-world problems. • compare circular functions and the trigonometric function values to draw inferences about coterminal angles and co-functions.
M.O.T.3.2	convert angle measures from degrees to radians (and vice versa) and apply this concept to • create a data set, analyze, and formulate a hypotheses to test and develop formulas for the arclength, area of a sector, and angular velocity and use the formula for application in the real-world. • compare and contrast the concepts of angular velocity and linear velocity and demonstrate by graphical or algebraic means relationship between them and apply to real-world problems.
M.O.T.3.3	using various methods, basic identities and graphical representation • verify trigonometric identities • prove the sum and difference to two angles, double-angles, and half-angle identities
M.O.T.3.4	justify and present the solutions of trigonometric equations that include both infinite and finite (over a restricted domain) solutions.
M.O.T.3.5	find the value of the inverse trigonometric functions using special angle trigonometric function values and technology. • draw inferences of restricted domain to recognize and produce a graph of the inverse trigonometric functions. • prove conjectures made about the solution of the equations such as $x = \sin(\arcsin y)$, $x = \sin(\arccos y)$ being sure to consider restrictions of the domain.
M.O.T.3.6	identify a real life problem utilizing graphs of trigonometric functions and/or the inverse functions; make a hypothesis as to the outcome; develop, justify, and implement a method to collect, organize, and analyze data;

	generalize the results to make a conclusion; compare the hypothesis and the conclusion; present the project using words, graphs, drawings, models, or tables.
M.O.T.3.7	model periodic data sets using graphs, tables, and equations and use them to analyze real-world problems such as electricity and harmonic motion.
M.O.T.3.8	investigate real-world problems within a project based investigation involving triangles using the trigonometric functions, the law of sines and the law of cosines, justify and present results.
M.O.T.3.9	develop and test a hypothesis to find the area of a triangle given the measures of two sides and the included angle or the measures of three sides (Heron's formula) and use these formulas to find total area of figures constructed of multiple shapes.
M.O.T.3.10	express complex numbers in polar form: • perform operations including adding, subtracting, multiplying, and dividing; • evaluate powers and roots of complex numbers using De Moivre's Theorem; and graph complex numbers. • graph complex numbers in the polar coordinate plane and make conjectures about some polar graphs and real-world situations such as the paths that the planets travel.
M.O.T.3.11	create graphical and algebraic representations for performing vector operations and analyze these to solve real-world problems such as force analysis and navigation.

Probability and Statistics Content Standards and Objectives

Probability and Statistics is one of the most important branches of the mathematical sciences. Knowledge of these topics is critical to decision-making and to the analysis of data. Using concepts of probability and statistics, individuals are able to predict the likelihood of an event occurring, organize and evaluate data, and identify the significance of statements. Connections between content and applications to the real-world will be emphasized. Graphing utilities such as calculators and computers will be used to enhance student learning and to aid in the solution of practical problems. Prerequisites for this course are successful completion of Algebra II and Geometry. The West Virginia Standards for 21st Century Learning include the following components: 21st Century Content Standards and Objectives and 21st Century Learning Skills and Technology Tools. All West Virginia teachers are responsible for classroom instruction that integrates learning skills, technology tools and content standards and objectives.

Grade 9-12	**Mathematics: Probability and Statistics**
Standard 5	Data Analysis and Probability
M.S.PS.5	Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will • formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them, • select and use appropriate statistical methods to analyze data, • develop and evaluate inferences and predictions that are based on models, and • apply and demonstrate an understanding of basic concepts of probability.

Performance Descriptors (M.PD.PS.3)				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Probability and Statistics students at the distinguished level: distinguish between, justify and investigate types of probability using multiple counting principles and distributions;	Probability and Statistics students at the above mastery level: distinguish between, justify and investigate types of probability using multiple counting principles and distributions;	Probability and Statistics students at the mastery level: distinguish between types of probability using multiple counting principles and distributions;	Probability and Statistics students at the partial mastery level: calculate probabilities given the type using multiple counting principles and distributions;	Probability and Statistics students at the novice level: recognize the types of probability using multiple counting principles and distributions;

use proper sampling techniques to compare and contrast more than one set of data that they collect, summarize, and interpret numerically and graphically in both one-variable and two-variable situations; test the validity of a hypothesis in real-world situations by determining the appropriate inference technique to make a conclusion about the population of interest	use proper sampling techniques to collect, summarize, and interpret data numerically and graphically in both one-variable and two-variable situations; test the validity of a hypothesis in real-world situations by determining the appropriate inference technique to make a conclusion about the population of interest	use proper sampling techniques to collect, summarize, and interpret data numerically and graphically in both one-variable and two-variable situations; test the validity of a hypothesis in real-world situations by determining the appropriate inference technique to make a conclusion about the population of interest.	use proper sampling techniques to collect and summarize data numerically and graphically in both one-variable and two-variable situations; test the validity of a hypothesis in real-world situations using the provided inference technique to make a conclusion about the population of interest.	collect and summarize data numerically and graphically in both one-variable and two-variable situation; identify a hypothesis in real-world situations to recognize that an inference technique needs to be used in order to make a conclusion about the population of interest.

Objectives	**Students will**
M.O.PS.5.1	distinguish between experimental and theoretical probability.
M.O.PS.5.2	using a real-world problem solving investigation, create and interpret data using various methods of displaying circle graphs, histograms, and frequency curves, make predictions, include information concerning outliers, present and justify results.
M.O.PS.5.3	determine possible outcomes using tree diagrams and the counting principles of permutations and combinations.
M.O.PS.5.4	express the chances of events occurring either in terms of a probability or odds.
M.O.PS.5.5	use the normal distribution and the binomial distribution including Pascal's triangle, to determine probability of events.
M.O.PS.5.6	analyze measures of central tendency (mean, median, and mode) from data presented in a variety of forms such as charts, tables, and graphs or from data created through experimentation.
M.O.PS.5.7	interpret and calculate measures of dispersions (range and standard deviation) from data presented in a variety of forms such as charts, tables and graphs or from data created through experimentation.

M.O.PS.5.8	analyze individual performances in terms of percentiles, z-scores, and t- scores.
M.O.PS.5.9	analyze the role of sampling, randomness, bias, and sample size in data collection and interpretation.
M.O.PS.5.10	identify a real life situation that involves statistical concepts including a t-test, make a hypothesis as to the outcome; develop, justify, and implement a method to collect, organize and analyze data; generalize the results to make a conclusion, compare the hypothesis and the conclusion; present the project using predictive and analytic tools (with and without technology).
M.O.PS.5.11	determine the correlation values for given data or for data generated by students and use the results to describe the association of the variables within the given data. Identify whether this association is systematic or predictable.
M.O.PS.5.12	calculate the Chi-Square values for a given population.
M.O.PS.5.13	perform a regression analysis on a set of data, either given or created through experimentation, and use the results to predict specific values of a variable. Identify the regression equation.
M.O.PS.5.14	perform an analysis of variance (ANOVA) and interpret the results.

Pre-Calculus Content Standards and Objectives

Pre-Calculus objectives extend students' knowledge of functions and equations (e.g., higher-order functions, exponential, and logarithmic) as well as provide preparation for a calculus course. Available technology will be used by students and teachers to enhance learning. Graphing utilities are powerful tools for solving and verifying equations and inequalities. They also aid in investigating functions, and their inverses. The West Virginia Standards for 21st Century Learning include the following components: 21st Century Content Standards and Objectives and 21st Century Learning Skills and Technology Tools. All West Virginia teachers are responsible for classroom instruction that integrates learning skills, technology tools, and content standards and objectives.

Grade 9-12	**Mathematics: Pre-Calculus**
Standard 2	Algebra
M.S.PC.2	Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will • demonstrate understanding of patterns, relations, and functions, • represent and analyze mathematical situations and structures using algebraic symbols, • use mathematical models to represent and understand quantitative relationships, and • analyze change in various contexts.

Performance Descriptors (M.PD.PC.2)				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Pre-Calculus students at the distinguished level: investigate and formulate a solution to a real-world problem involving higher-order polynomials, exponential and logarithmic equations; solve application problems involving	Pre-Calculus students at the above mastery level: hypothesize and develop a plan to solve higher-order polynomials, exponential and logarithmic equations; solve application	Pre-Calculus students at the mastery level: determine the reasonableness of the solutions of higher-order polynomials, exponential and logarithmic equations; evaluate sequences and series to find or estimate a limit;	Pre-Calculus students at the partial mastery level: examine the solutions of higher-order polynomials, exponential and logarithmic equations; differentiate sequences	Pre-Calculus students at the novice level: confirm the solutions of higher-order polynomials, exponential and logarithmic equations; recognize sequences and series, and find or

sequences and series, and formulate a hypothesis for evaluating limits; consider and justify using induction to prove formulas and statements; differentiate between appropriate methods to expand binomials in relation to real-world problems; compare and contrast various graphs formulating a set of rules that produce and support a solution to a real-world problem; design and execute a method to solve a real-world problem involving vectors.	problems involving sequences and series and evaluate limits; devise a method for proving formulas and statements; expand binomials by applying appropriate methods and relate the expansion to real-world situations; identify and justify their solutions to real-world problems which require various graphs; recognize the application of vectors to practical problems and perform operations on vectors to solve them.	differentiate the process of proving formulas and statements; expand binomials by applying appropriate methods; interpret the techniques of curve sketching to graph functions of real-world situations; analyze and perform operations on vectors to solve practical problems.	and series, and find or estimate a limit; find formulas and statements by applying induction; identify the various methods for expanding binomials; graph various functions; perform operations on vectors to solve practical problems.	estimate a limit; recognize induction as a process to prove statements and formulas; recognize the methods for expanding binomials; identify the graphs of various functions; perform operations on vectors.

Objectives	**Students will**
M.O.PC.2.1	investigate and sketch the graphs of polynomials and rational functions by analyzing and using the characteristics of zeros, upper and lower bounds, y-intercepts, symmetry, asymptotes and end behavior, maximum and minimum points, and domain and range.
M.O.PC.2.2	solve higher order polynomial equations utilizing techniques such as Descartes' Rule of Signs, upper and lower bounds, and the Rational Root Theorem.

M.O.PC.2.3	relate Pascal's Triangle and the Binomial Theorem; use both to expand binomials with positive integral exponents.
M.O.PC.2.4	establish and explain the inverse relationship between exponential and logarithmic functions; graph related functions and include their domain and range using interval notation.
M.O.PC.2.5	compare laws of exponents to properties of logarithms; solve equations and practical problems involving exponential and logarithmic expressions, including natural and common logarithms; confirm solutions graphically and numerically.
M.O.PC.2.6	solve problems involving the sum of finite and infinite sequences and series, including Sigma notation.
M.O.PC.2.7	use tables of values, graphs, conjectures, algebraic methods, and numerical substitution to find or estimate the limit of a function, a sequence or a series.
M.O.PC.2.8	analyze and describe the geometry of vectors, perform mathematical operations with vectors and use vectors to solve practical problems.
M.O.PC.2.9	apply the method of mathematical induction to prove formulas and statements.
M.O.PC.2.10	apply parametric methods to represent motion of objects.
M.O.PC.2.11	use multiple representations, such as words, graphs, tables, and equations, to solve practical problems involving logarithmic, exponential, polynomial, rational, and radical functions; explain how the representations are related to each other, as well as to the problem.

Grade 9-12	**Mathematics: Pre-Calculus**			
Standard 3	Geometry			
M.S.PC.3	Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will • analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships, • specify locations and describe spatial relationships using coordinate geometry and other representational systems, • apply transformations and use symmetry to analyze mathematical situations, and • solve problems using visualization, spatial reasoning, and geometric modeling.			
Performance Descriptors (M.PD.PD.3)				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice

Pre-Calculus students at the distinguished level: hypothesize, organize, determine and explain the justification for the solutions to real-world problems involving conic sections and their transformations.	Pre-Calculus students at the above mastery level: analyze, interpret, and graph the conic sections along with their transformations, and apply to real-world situations.	Pre-Calculus students at the mastery level: analyze, interpret, and graph conic sections and their transformations.	Pre-Calculus students at the partial mastery level: graph conic sections and their transformations.	Pre-Calculus students at the novice level: identify the graphs of conic sections and their transformations.

Objectives	**Students will**
M.O.PC.3.1	graph functions and conic sections using transformations.
M.O.PC.3.2	analyze and describe properties of conic sections; explain the interrelationship among the properties; solve practical problems involving conic sections.

Grade 9-12	**Mathematics: Pre-Calculus**
Standard 5	Data Analysis and Probability
M.S.PC.5	Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will • formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them, • select and use appropriate statistical methods to analyze data, • develop and evaluate inferences and predictions that are based on models, and • apply and demonstrate an understanding of basic concepts of probability.

Performance Descriptors (M.PD.PC.5)

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Pre-Calculus students at the distinguished level: relate and defend a solution to a developed real-world situation that involves use of	Pre-Calculus students at the above mastery level: summarize the analysis of developed regression equations.	Pre-Calculus students at the mastery level: investigate, hypothesize, and develop a regression equation.	Pre-Calculus students at the partial mastery level: investigate and hypothesize regarding a regression equation.	Pre-Calculus students at the novice level: investigate and select a regression equation.

regression equations.				

Objectives	**Students will**
M.O.PC.5.1	identify a real life situation that exhibits characteristics of exponential or logistic growth or decay; pose a question; make a hypothesis as to the answer; develop, justify, and implement a method to collect, organize, and analyze related data; extend the nature of collected, discrete data to that of a continuous function that describes the known data set; generalize the results to make a conclusion; compare the hypothesis and the conclusion; present the project numerically, analytically, graphically and verbally using the predictive and analytic tools of pre-calculus (with and without technology).

Calculus Content Standards and Objectives

Calculus objectives are designed for students who have completed Algebra I, Geometry, Algebra II, Trigonometry, and Pre-Calculus. Study includes functions and continuity, limits, differentiation and applications of derivatives, integration and its application to area, volume, and displacement. The Rule of Four (Numerical, Analytical, Graphical and Verbal) will be applied throughout the course. Available technology will be used by students and teachers to enhance learning. Graphing utilities will be used to investigate concepts and to evaluate derivatives and integrals. The West Virginia Standards for 21st Century Learning include the following components: 21st Century Content Standards and Objectives and 21st Century Learning Skills and Technology Tools. All West Virginia teachers are responsible for classroom instruction that integrates learning skills, technology tools, and content standards and objectives.

Grade 9-12	**Mathematics: Calculus**
Standard 2	Algebra
M.S.C.2	Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will • demonstrate understanding of patterns, relations, and functions, • represent and analyze mathematical situations and structures using algebraic symbols, • use mathematical models to represent and understand quantitative relationships, and • analyze change in various contexts.

Performance Descriptors (M.PD.C.2)				
Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Calculus students at the distinguished level: closely connect all representations of a function; recognize real life situations that involve limits and interpret	Calculus students at the above mastery level: explain connections among algebraic notation, graphical analysis and tabular data; explain limits using multiple representations	Calculus students at the mastery level: manipulate algebraic notation to study functions and relate the results to graphs and tables; determine limits algebraically, graphically and	Calculus students at the partial mastery level: use algebraic notation for functions and confirm results using graphs; determine limits graphically or	Calculus students at the novice level: recognize functions expressed algebraically and graphically and use functional notation correctly; given a graph or table, recognize a limit and evaluate limits using

these limits using multiple representations and evaluate them using appropriate limit properties; relate the Intermediate Value Theorem, continuity, and root finding; apply the various forms of the definition of the derivative of a function at a point; interpreted as the slope of the tangent line to the graph of the function at any x, and as the instantaneous rate of change. They recognize the tangent line slope as a limit of the converging secant line slopes, and apply the limit definition to find a general form for f'(x); compare the average rate of change and the instantaneous rate of change in real-world applications. They	and evaluate limits using appropriate limit properties; determine if a function is continuous at a point over an interval; apply the definition of the derivative of a function at a point; interpret this as the slope of the tangent line and as the instantaneous rate of change. They recognize the tangent line slope as a limit of the converging secant line slopes and apply the limit definition to find a general form for f'(x); compare the average rate of change and the instantaneous rate of change in real-world	numerically, using appropriate limit properties; decide about continuity at a point and over an interval; interpret the derivative of a function as the slope of the tangent line to the graph of the function at any x, or as the instantaneous rate of change. They apply the limit definition to find the derivative at a point; investigate the average rate of change and instantaneous rate of change in real-world	numerically and evaluate limits using limit properties; determine if a function is continuous at a point numerically and graphically; apply the definition of the derivative of a function at a point to find the slope of the tangent line to the graph of the function, interpreting the derivative as an instantaneous rate of change; investigate the average rate of change and	limit properties; determine graphically if a function is continuous at a point; construct the tangent line to a curve at a given point and use derivatives to aid in graphing functions; calculate the average rate of change and the instantaneous rate of change;

prove that differentiability implies continuity and give examples of continuous functions that are not differentiable. They combine and apply the rules of differentiation to various types of functions as appropriate. They use Rolle's Theorem to derive the Mean Value Theorem; use mathematical models to solve applied problems; efficiently calculate indefinite or definite integrals. They calculate a definite integral of a polynomial function using an infinite limit of a Riemann sum and apply the Fundamental Theorem of Calculus.	applications, demonstrate that differentiability implies continuity, and give examples of continuous functions that are not differentiable. They combine and apply the rules of differentiation to various types of functions as appropriate. They recognize when the hypotheses of Rolle's and the Mean Value Theorems are satisfied; construct and apply mathematical models to solve applied problems; find definite and indefinite integrals that may involve multiple substitutions and change of limits and calculate a definite integral of a polynomial function using an infinite limit of a Riemann sum. They apply the Fundamental Theorem of Calculus.	applications. They relate differentiability and continuity and combine and apply the algebraic rules of differentiation and theoretical results; construct and apply mathematical models to solve applied problems; calculate definite and indefinite integrals for integrable elementary functions. They calculate definite integrals using both Riemann sums and the Fundamental Theorem of Calculus.	instantaneous rate of change graphically. They recognize that differentiable functions are also continuous. They apply the rules of differentiation to various types of functions; solve applied problems about motion, area, and volume; calculate definite and indefinite integrals for integrable elementary functions. They apply the Fundamental Theorem of Calculus to evaluate a definite integral.	solve simple optimization problems; calculate definite and indefinite integrals for polynomials. They apply the Fundamental Theorem of Calculus to evaluate a definite integral.

Objectives	Students will
M.O.C.2.1	use abstract notation to apply properties of algebraic, trigonometric, exponential, logarithmic and composite functions, as well as their inverses, represented graphically, numerically, analytically, and verbally; and demonstrate an understanding of the connections among these representations.
M.O.C.2.2	demonstrate a conceptual understanding of the definition of a limit via the analysis of continuous and discontinuous functions represented using multiple representations (e.g. graphs and tables).
M.O.C.2.3	$\lim_{x\to 0}\frac{1-\cos x}{x}=0$ use the properties of limits including addition, product, quotient, composition, and squeeze/sandwich theorem to calculate the various forms of limits: one-sided limits, limits at infinity, infinite limits, limits that do not exist, and special limits such as $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1$
M.O.C.2.4	apply the definition of continuity to determine where a function is continuous or discontinuous including continuity at a point, continuity over an interval, application of the Intermediate Value Theorem, and graphical interpretation of continuity and discontinuity.
M.O.C.2.5	investigate and apply the definition of the derivative graphically, numerically, and analytically at a point, conceptually interpreting the derivative as an instantaneous rate of change and the slope of the tangent line.
M.O.C.2.6	discriminate between the average rate of change and the instantaneous rate of change using real-world problems.
M.O.C.2.7	justify why differentiability implies continuity and classify functional cases when continuity does not imply differentiability.
M.O.C.2.8	recognize when the Extreme Value Theorem indicates that function extrema exist.
M.O.C.2.9	quickly recall and apply rules of differentiation including the constant multiple rule, sum rule, the difference rule, the product rule, the quotient rule, the power rule, and the chain rule as applied to algebraic, trigonometric, exponential, logarithmic, and inverse trigonometric functions using techniques of both explicit and implicit differentiation.
M.O.C.2.10	apply Rolle's Theorem and the Mean Value Theorem to real-world problems.
M.O.C.2.11	construct and use mathematical models to solve optimization, related-rates, velocity, and acceleration problems.
M.O.C.2.12	determine antiderivatives that follow from derivatives of basic functions and apply substitution of variables.
M.O.C.2.13	calculate a definite integral using Riemann sums by evaluating an infinite limit of a sum using summation notation and rules for summation.
M.O.C.2.14	evaluate definite integrals using basic integration properties such as addition, subtraction, constant multipliers, the

	power rule, substitution, and change of limits.
M.O.C.2.15	characterize the definite integral as the total change of a function over an interval and use this to solve real-world problems.
M.O.C.2.16	apply the Fundamental Theorem of Calculus to evaluate definite integrals and to formulate a cumulative area function and interpret the function as it relates to the integrand.
M.O.C.2.17	use integration to solve problems that involve linear displacement, total distance, position, velocity, acceleration and area between curves by looking at both functions of x and functions of y; utilize units to interpret the physical nature of the calculus process.

Grade 9-12	**Mathematics: Calculus**
Standard 3	Geometry
M.S.C.3	Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will • analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships, • specify locations and describe spatial relationships using coordinate geometry and other representational systems, • apply transformations and use symmetry to analyze mathematical situations, and • solve problems using visualization, spatial reasoning, and geometric modeling.

Performance Descriptors (M.PD.C.3)

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Calculus students at the distinguished level: apply the definition of continuity to categorize discontinuities of functions presented algebraically and graphically; use asymptotes to explain end behavior of	Calculus students at the above mastery level: recognize continuous and discontinuous functions using limits; use limits to find and justify the existence of	Calculus students at the mastery level: recognize continuous and discontinuous functions graphically; apply limits to recognize asymptotes,	Calculus students at the partial mastery level: distinguish between continuous and discontinuous functions graphically; apply limits to find asymptotes, use a	Calculus students at the novice level: identify a discontinuous function graphically; given a graph, identify the location of

functions, and describe asymptotic behavior using multiple representations, develop tangent lines as best linear approximations to functions near specific points and apply this concept to Newton's Method; investigate and explain the relationships among the graphs of a function and its derivatives; anticipate whether the left, right, or midpoint rule will yield the best approximation to a definite integral using a Riemann Sum with a finite number of sub-intervals. They propose better methods for approximating the actual area.	asymptotes of functions, develop tangent lines as best linear approximations to functions near specific points, construct these tangent lines and apply this concept to Newton's Method; investigate and explain the relationships among the graph of a function and its derivatives; approximate the area under a curve using a Riemann sum implementing left, right, or midpoint rules, and determine whether the left hand and right hand approximations over-estimate or under-estimate the actual area.	use tangent lines to locally approximate functions, and apply Newton's Method to approximate zeroes of functions; extract information about the graph of a function from its derivative and limiting values; approximate the area under a curve via a Riemann sum using left, right, or midpoint rules.	tangent line to approximate a function at a point and can apply Newton's Method to approximate zeroes of functions; use derivatives to aid in graphing functions; approximate the area under a curve by constructing a Riemann sum implementing left, right, or midpoint rules.	asymptotes; use information from derivatives to aid in graphing functions; approximate the area under a curve by applying a finite Riemann sum implementing left, right, or midpoint rules, given the subdivision.

Objectives	**Students will**
M.O.C.3.1	use limits to deduce asymptotic behavior of the graph of a function.
M.O.C.3.2	compare and contrast the limit definition (not delta epsilon) of continuity and the graphical interpretation of the continuity of a function at a point; recognize different types of discontinuities.
M.O.C.3.3	develop tangent lines as best linear approximations to functions near specific points; explain this conceptually; and construct these tangent lines; and apply this concept to Newton's Method.

M.O.C.3.4	investigate and explain the relationships among the graphs of a function, its derivative and its second derivative; construct the graph of a function using the first and second derivatives including extrema, points of inflection, and asymptotic behavior.
M.O.C.3.5	approximate areas under a curve using Riemann sums by applying and comparing left, right, and midpoint methods for a finite number of subintervals.

Grade 9-12	**Mathematics: Calculus**
Standard 5	Data Analysis and Probability
M.S.C.5	Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will • formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them, • select and use appropriate statistical methods to analyze data, • develop and evaluate inferences and predictions that are based on models, and • apply and demonstrate an understanding of basic concepts of probability.

Performance Descriptors (M.PD.C.5)

Distinguished	Above Mastery	Mastery	Partial Mastery	Novice
Calculus students at the distinguished level: apportion individual tasks in small groups to identify a real life situation that involves modeling change; pose a question; make a hypothesis as to the answer; develop, justify, and implement a method to collect, organize, and analyze related data; extend the nature of collected,	Calculus students at the above mastery level: in small groups, identify a real life situation that involves modeling change; pose a question; make a hypothesis as to the answer; implement a method to collect, organize, and analyze related data; extend the nature of collected, discrete data to that of a continuous function that	Calculus students at the mastery level: working in small groups, identify a real life situation that involves modeling change; pose a question; implement a method to collect, organize, and analyze related data; find a continuous function that describes the known data set; make predictions to test their	Calculus students at the partial mastery level: working in teacher facilitated groups, solve a real life problem using given data that involves modeling change. They organize and analyze the data; find a continuous function that describes the known data set. They collaborate using concepts from calculus to present the projects	Calculus students at the novice level: working in teacher facilitated groups, solve a real life problem using provided data that involves modeling change. They extend collected, discrete data to values of a continuous function that describes the known data set. They present the projects numerically,

discrete data to that of a continuous function that describes the known data set; generalize the results to make predictions to test their model; compare the hypothesis and the conclusion. They present the project numerically, analytically, graphically and verbally.	describes the known data set; generalize the results to make predictions to test their model; compare the hypothesis and the conclusion. They collaborate using concepts from calculus to present the project numerically, analytically, graphically and verbally.	model. They collaborate using concepts from calculus to present the project numerically, analytically, graphically and verbally.	numerically, analytically, graphically and verbally.	analytically, graphically and verbally.

Objectives	**Students will**
M.O.C.5.1	identify a real life situation that involves quantities that change over time; pose a question; make a hypothesis as to the answer; develop, justify, and implement a method to collect, organize, and analyze related data; extend the nature of collected, discrete data to that of a continuous function that describes the known data set; generalize the results to make a conclusion; compare the hypothesis and the conclusion; present the project numerically, analytically, graphically and verbally using the predictive and analytic tools of calculus.